In each of Problems 13 through 20, find the solution of the given initial value problem.

13. $y' - y = 2te^{2t}, \qquad y(0) = 1$
14. $y' + 2y = te^{-2t}, \qquad y(1) = 0$
15. $ty' + 2y = t^2 - t + 1, \qquad y(1) = \frac{1}{2}, \quad t > 0$
16. $y' + (2/t)y = (\cos t)/t^2, \qquad y(\pi) = 0, \quad t > 0$
17. $y' - 2y = e^{2t}, \qquad y(0) = 2$
18. $ty' + 2y = \sin t, \qquad y(\pi/2) = 1, \quad t > 0$
19. $t^3y' + 4t^2y = e^{-t}, \qquad y(-1) = 0, \quad t < 0$
20. $ty' + (t+1)y = t, \qquad y(\ln 2) = 1, \quad t > 0$

In each of Problems 1 through 8, solve the given differential equation.

1. $y' = x^2/y$
2. $y' = x^2/y(1+x^3)$
3. $y' + y^2 \sin x = 0$
4. $y' = (3x^2 - 1)/(3 + 2y)$
5. $y' = (\cos^2 x)(\cos^2 2y)$
6. $xy' = (1 - y^2)^{1/2}$
7. $\dfrac{dy}{dx} = \dfrac{x - e^{-x}}{y + e^y}$
8. $\dfrac{dy}{dx} = \dfrac{x^2}{1 + y^2}$

In each of Problems 9 through 20:

(a) Find the solution of the given initial value problem in explicit form.

9. $y' = (1 - 2x)y^2, \qquad y(0) = -1/6$
10. $y' = (1 - 2x)/y, \qquad y(1) = -2$
11. $x\,dx + ye^{-x}dy = 0, \qquad y(0) = 1$
12. $dr/d\theta = r^2/\theta, \qquad r(1) = 2$
13. $y' = 2x/(y + x^2y), \qquad y(0) = -2$
14. $y' = xy^3(1 + x^2)^{-1/2}, \qquad y(0) = 1$
15. $y' = 2x/(1 + 2y), \qquad y(2) = 0$
16. $y' = x(x^2 + 1)/4y^3, \qquad y(0) = -1/\sqrt{2}$
17. $y' = (3x^2 - e^x)/(2y - 5), \qquad y(0) = 1$
18. $y' = (e^{-x} - e^x)/(3 + 4y), \qquad y(0) = 1$
19. $\sin 2x\,dx + \cos 3y\,dy = 0, \qquad y(\pi/2) = \pi/3$
20. $y^2(1 - x^2)^{1/2}dy = \arcsin x\,dx, \qquad y(0) = 1$

Determine whether each of the equations in Problems 1 through 12 is exact. If it is exact, find the solution.

1. $(2x + 3) + (2y - 2)y' = 0$
2. $(2x + 4y) + (2x - 2y)y' = 0$
3. $(3x^2 - 2xy + 2) + (6y^2 - x^2 + 3)y' = 0$
4. $(2xy^2 + 2y) + (2x^2y + 2x)y' = 0$
5. $\dfrac{dy}{dx} = -\dfrac{ax + by}{bx + cy}$
6. $\dfrac{dy}{dx} = -\dfrac{ax - by}{bx - cy}$
7. $(e^x \sin y - 2y \sin x) + (e^x \cos y + 2\cos x)y' = 0$
8. $(e^x \sin y + 3y) - (3x - e^x \sin y)y' = 0$
9. $(ye^{xy} \cos 2x - 2e^{xy} \sin 2x + 2x) + (xe^{xy} \cos 2x - 3)y' = 0$
10. $(y/x + 6x) + (\ln x - 2)y' = 0, \qquad x > 0$
11. $(x \ln y + xy) + (y \ln x + xy)y' = 0; \qquad x > 0, \quad y > 0$
12. $\dfrac{x}{(x^2 + y^2)^{3/2}} + \dfrac{y}{(x^2 + y^2)^{3/2}} \dfrac{dy}{dx} = 0$

## *Bernoulli Differential Equations*

**2.72** $3xy' - 3xy^4 \ln x - y = 0$ **Ans** $\dfrac{1}{y^3} = -\dfrac{3}{4}x(2\ln x - 1) + \dfrac{C}{x};\ \ y = 0$

**2.73** $\dfrac{dy}{dx} = \dfrac{4x^3y^2}{x^4y + 2}$ **Ans** $x^4 = -\dfrac{1}{y} + Cy;\ \ y = 0$

**2.74** $y(6y^2 - x - 1)\,dx + 2x\,dy = 0$ **Ans** $\dfrac{1}{y^2} = \dfrac{1}{x}(6 + Ce^{-x});\ \ y = 0$

**2.75** $(1 + x)(y' + y^2) - y = 0$ **Ans** $\dfrac{1}{y} = \dfrac{1}{1+x}\left(\dfrac{x^2}{2} + x + C\right);\ \ y = 0$

**2.76** $xyy' + y^2 - \sin x = 0$ **Ans** $x^2y^2 = -2x\cos x + 2\sin x + C$

**2.77** $(2x^3 - y^4)\,dx + xy^3\,dy = 0$ **Ans** $y^4 = 8x^3 + Cx^4$

**2.78** $y' - y\tan x + y^2\cos x = 0$ **Ans** $\dfrac{1}{y} = \cos x\,(x + C);\ \ y = 0$

**2.79** $6y^2\,dx - x(2x^3 + y)\,dy = 0$ **Ans** $(y - 2x^3)^2 = Cyx^6;\ \ y = 0$

## *Exact Differential Equations*

**2.20** $(3x^2 + 6xy^2)\,dx + (6x^2y + 4y^3)\,dy = 0$ **Ans** $x^3 + 3x^2y^2 + y^4 = C$

**2.21** $(2x^3 - xy^2 - 2y + 3)\,dx - (x^2y + 2x)\,dy = 0$

**Ans** $x^4 - x^2y^2 - 4xy + 6x = C$

**2.22** $(xy^2 + x - 2y + 3)\,dx + x^2y\,dy = 2(x + y)\,dy$

**Ans** $x^2y^2 + x^2 + 6x - 4xy - 2y^2 = C$

**2.23** $3y(x^2 - 1)\,dx + (x^3 + 8y - 3x)\,dy = 0,$ when $x = 0,\ y = 1$

**Ans** $x^3y - 3xy + 4y^2 = 4$

**2.24** $(x^2 + \ln y)\,dx + \dfrac{x}{y}\,dy = 0$ **Ans** $\frac{1}{3}x^3 + x\ln y = C$

**2.25** $2x(3x + y - ye^{-x^2})\,dx + (x^2 + 3y^2 + e^{-x^2})\,dy = 0$

**Ans** $2x^3 + x^2y + ye^{-x^2} + y^3 = C$

**2.26** $(3 + y + 2y^2\sin^2 x)\,dx + (x + 2xy - y\sin 2x)\,dy = 0$

**Ans** $3x + xy + xy^2 - \frac{1}{2}y^2\sin 2x = C$

**2.27** $(2xy + y^2)\,dx + (x^2 + 2xy + y^2)\,dy = 0$ **Ans** $x^2y + xy^2 + \frac{1}{3}y^3 = C$

### *Variable Separable*

**2.1** $\cos^2 y\,dx+(1+e^{-x})\sin y\,dy=0$ **ANS** $\ln(e^x+1)=-\dfrac{1}{\cos y}+C;\ \cos y=0$

**2.2** $\dfrac{dy}{dx}=\dfrac{x^3e^{x^2}}{y\ln y}$ **ANS** $y^2\left(\ln y-\frac{1}{2}\right)=e^{x^2}(x^2-1)+C$

**2.3** $x\cos^2 y\,dx+e^x\tan y\,dy=0$ **ANS** $e^{-x}(x+1)=\dfrac{1}{2\cos^2 y}+C;\ \cos y=0$

**2.4** $x\,(y^2+1)\,dx+(2y+1)\,e^{-x}\,dy=0$

**ANS** $(x-1)e^x+\ln(y^2+1)+\tan^{-1}y=C$

**2.5** $xy^3\,dx+e^{x^2}\,dy=0$ **ANS** $e^{-x^2}+\dfrac{1}{y^2}=C;\ y=0$

**2.6** $x\cos^2 y\,dx+\tan y\,dy=0$ **ANS** $x^2+\tan^2 y=C$

**2.7** $xy^3\,dx+(y+1)e^{-x}\,dy=0$ **ANS** $e^x(x-1)-\dfrac{1}{y}-\dfrac{1}{2y^2}=C;\ y=0$

### *Homogeneous and Special Transformations*

**2.8** $\dfrac{dy}{dx}+\dfrac{x}{y}+2=0$ **ANS** $\ln|x+y|+\dfrac{x}{x+y}=C;\ y=-x$

**2.9** $x\,dy-y\,dx=x\cot\left(\dfrac{y}{x}\right)dx$ **ANS** $\cos\left(\dfrac{y}{x}\right)=\dfrac{C}{x}$

**2.10** $\left[x\cos^2\left(\dfrac{y}{x}\right)-y\right]dx+x\,dy=0$ **ANS** $\ln|x|+\tan\dfrac{y}{x}=C;\ \cos\dfrac{y}{x}=0$

**2.11** $x\,dy=y(1+\ln y-\ln x)\,dx$ **ANS** $y=xe^{Cx}$

**2.12** $xy\,dx+(x^2+y^2)\,dy=0$ **ANS** $y^2(2x^2+y^2)=C$

**2.13** $\left[1+\exp\left(-\dfrac{y}{x}\right)\right]dy+\left(1-\dfrac{y}{x}\right)dx=0$ **ANS** $x\exp\left(\dfrac{y}{x}\right)+y=C$

**2.14** $(x^2-xy+y^2)\,dx-xy\,dy=0$ **ANS** $(y-x)e^{y/x}=C$

**2.15** $(3+2x+4y)y'=1+x+2y$

**ANS** $8y-4x+\ln|4x+8y+5|=C;\ \ 4x+8y+5=0$

**2.16** $y'=\dfrac{2x+y-1}{x-y-2}$ **ANS** $\sqrt{2}\tan^{-1}\dfrac{y+1}{\sqrt{2}(x-1)}=\ln\left[(y+1)^2+2(x-1)^2\right]+C$

**2.17** $(y+2)\,dx=(2x+y-4)dy$ **ANS** $(y+2)^2=C(x+y-1);\ \ y=1-x$

**2.18** $y'=\sin^2(x-y)$ **ANS** $x=\tan(x-y)+C;\ x-y=\dfrac{\pi}{2}\pm k\pi,\ k=0,1,2,\ldots$

**2.19** $\dfrac{dy}{dx}=(x+1)^2+(4y+1)^2+8xy+1$ **ANS** $\dfrac{2}{3}(x+4y+1)=\tan(6x+C)$