In each of Problems 1 through 8, find the general solution of the given differential equation.

1. $y'' + 2y' - 3y = 0$
2. $y'' + 3y' + 2y = 0$
3. $6y'' - y' - y = 0$
4. $2y'' - 3y' + y = 0$
5. $y'' + 5y' = 0$
6. $4y'' - 9y = 0$
7. $y'' - 9y' + 9y = 0$
8. $y'' - 2y' - 2y = 0$

In each of Problems 9 through 16, find the solution of the given initial value problem. Sketch the graph of the solution and describe its behavior as t increases.

9. $y'' + y' - 2y = 0, \quad y(0) = 1, \quad y'(0) = 1$
10. $y'' + 4y' + 3y = 0, \quad y(0) = 2, \quad y'(0) = -1$
11. $6y'' - 5y' + y = 0, \quad y(0) = 4, \quad y'(0) = 0$
12. $y'' + 3y' = 0, \quad y(0) = -2, \quad y'(0) = 3$
13. $y'' + 5y' + 3y = 0, \quad y(0) = 1, \quad y'(0) = 0$
14. $2y'' + y' - 4y = 0, \quad y(0) = 0, \quad y'(0) = 1$
15. $y'' + 8y' - 9y = 0, \quad y(1) = 1, \quad y'(1) = 0$
16. $4y'' - y = 0, \quad y(-2) = 1, \quad y'(-2) = -1$
17. Find a differential equation whose general solution is $y = c_1e^{2t} + c_2e^{-3t}$.
18. Find a differential equation whose general solution is $y = c_1e^{-t/2} + c_2e^{-2t}$.

In each of Problems 1 through 6, find the Wronskian of the given pair of functions.

1. $e^{2t}, \quad e^{-3t/2}$
2. $\cos t, \quad \sin t$
3. $e^{-2t}, \quad te^{-2t}$
4. $x, \quad xe^x$
5. $e^t \sin t, \quad e^t \cos t$
6. $\cos^2 \theta, \quad 1 + \cos 2\theta$

In each of Problems 7 through 12, determine the longest interval in which the given initial value problem is certain to have a unique twice-differentiable solution. Do not attempt to find the solution.

7. $ty'' + 3y = t, \quad y(1) = 1, \quad y'(1) = 2$
8. $(t - 1)y'' - 3ty' + 4y = \sin t, \quad y(-2) = 2, \quad y'(-2) = 1$
9. $t(t - 4)y'' + 3ty' + 4y = 2, \quad y(3) = 0, \quad y'(3) = -1$
10. $y'' + (\cos t)y' + 3(\ln |t|)y = 0, \quad y(2) = 3, \quad y'(2) = 1$
11. $(x - 3)y'' + xy' + (\ln |x|)y = 0, \quad y(1) = 0, \quad y'(1) = 1$
12. $(x - 2)y'' + y' + (x - 2)(\tan x)y = 0, \quad y(3) = 1, \quad y'(3) = 2$
13. Verify that $y_1(t) = t^2$ and $y_2(t) = t^{-1}$ are two solutions of the differential equation $t^2y'' - 2y = 0$ for $t > 0$. Then show that $y = c_1t^2 + c_2t^{-1}$ is also a solution of this equation for any c_1 and c_2.
14. Verify that $y_1(t) = 1$ and $y_2(t) = t^{1/2}$ are solutions of the differential equation $yy'' + (y')^2 = 0$ for $t > 0$. Then show that $y = c_1 + c_2t^{1/2}$ is not, in general, a solution of this equation. Explain why this result does not contradict Theorem 3.2.2.
15. Show that if $y = \phi(t)$ is a solution of the differential equation $y'' + p(t)y' + q(t)y = g(t)$, where $g(t)$ is not always zero, then $y = c\phi(t)$, where c is any constant other than 1, is not a solution. Explain why this result does not contradict the remark following Theorem 3.2.2.
16. Can $y = \sin(t^2)$ be a solution on an interval containing $t = 0$ of an equation $y'' + p(t)y' + q(t)y = 0$ with continuous coefficients? Explain your answer.

17. If the Wronskian W of f and g is $3e^{4t}$, and if $f(t) = e^{2t}$, find $g(t)$.

18. If the Wronskian W of f and g is t^2e^t, and if $f(t) = t$, find $g(t)$.

19. If $W(f, g)$ is the Wronskian of f and g, and if $u = 2f - g$, $v = f + 2g$, find the Wronskian $W(u, v)$ of u and v in terms of $W(f, g)$.

20. If the Wronskian of f and g is $t\cos t - \sin t$, and if $u = f + 3g$, $v = f - g$, find the Wronskian of u and v.

21. Assume that y_1 and y_2 are a fundamental set of solutions of $y'' + p(t)y' + q(t)y = 0$ and let $y_3 = a_1y_1 + a_2y_2$ and $y_4 = b_1y_1 + b_2y_2$, where a_1, a_2, b_1, and b_2 are any constants. Show that

$$W(y_3, y_4) = (a_1b_2 - a_2b_1)W(y_1, y_2).$$

Are y_3 and y_4 also a fundamental set of solutions? Why or why not?

In each of Problems 22 and 23, find the fundamental set of solutions specified by Theorem 3.2.5 for the given differential equation and initial point.

22. $y'' + y' - 2y = 0, \qquad t_0 = 0$

23. $y'' + 4y' + 3y = 0, \qquad t_0 = 1$

In each of Problems 24 through 27, verify that the functions y_1 and y_2 are solutions of the given differential equation. Do they constitute a fundamental set of solutions?

24. $y'' + 4y = 0; \qquad y_1(t) = \cos 2t, \quad y_2(t) = \sin 2t$

25. $y'' - 2y' + y = 0; \qquad y_1(t) = e^t, \quad y_2(t) = te^t$

26. $x^2y'' - x(x + 2)y' + (x + 2)y = 0, \quad x > 0; \qquad y_1(x) = x, \quad y_2(x) = xe^x$

27. $(1 - x\cot x)y'' - xy' + y = 0, \quad 0 < x < \pi; \qquad y_1(x) = x, \quad y_2(x) = \sin x$

In each of Problems 1 through 6, use Euler's formula to write the given expression in the form $a + ib$.

1. $\exp(1 + 2i)$
2. $\exp(2 - 3i)$
3. $e^{i\pi}$
4. $e^{2-(\pi/2)i}$
5. 2^{1-i}
6. π^{-1+2i}

In each of Problems 7 through 16, find the general solution of the given differential equation.

7. $y'' - 2y' + 2y = 0$
8. $y'' - 2y' + 6y = 0$
9. $y'' + 2y' - 8y = 0$
10. $y'' + 2y' + 2y = 0$
11. $y'' + 6y' + 13y = 0$
12. $4y'' + 9y = 0$
13. $y'' + 2y' + 1.25y = 0$
14. $9y'' + 9y' - 4y = 0$
15. $y'' + y' + 1.25y = 0$
16. $y'' + 4y' + 6.25y = 0$

In each of Problems 17 through 22, find the solution of the given initial value problem. Sketch the graph of the solution and describe its behavior for increasing t.

17. $y'' + 4y = 0, \qquad y(0) = 0, \quad y'(0) = 1$

18. $y'' + 4y' + 5y = 0, \qquad y(0) = 1, \quad y'(0) = 0$

19. $y'' - 2y' + 5y = 0, \qquad y(\pi/2) = 0, \quad y'(\pi/2) = 2$

20. $y'' + y = 0, \qquad y(\pi/3) = 2, \quad y'(\pi/3) = -4$

21. $y'' + y' + 1.25y = 0, \qquad y(0) = 3, \quad y'(0) = 1$

22. $y'' + 2y' + 2y = 0, \qquad y(\pi/4) = 2, \quad y'(\pi/4) = -2$

In each of Problems 1 through 10, find the general solution of the given differential equation.

1. $y'' - 2y' + y = 0$
2. $9y'' + 6y' + y = 0$
3. $4y'' - 4y' - 3y = 0$
4. $4y'' + 12y' + 9y = 0$
5. $y'' - 2y' + 10y = 0$
6. $y'' - 6y' + 9y = 0$

In each of Problems 11 through 14, solve the given initial value problem.

11. $9y'' - 12y' + 4y = 0, \quad y(0) = 2, \quad y'(0) = -1$
12. $y'' - 6y' + 9y = 0, \quad y(0) = 0, \quad y'(0) = 2$
13. $9y'' + 6y' + 82y = 0, \quad y(0) = -1, \quad y'(0) = 2$
14. $y'' + 4y' + 4y = 0, \quad y(-1) = 2, \quad y'(-1) = 1$

In each of Problems 23 through 30, use the method of reduction of order to find a second solution of the given differential equation.

23. $t^2y'' - 4ty' + 6y = 0, \quad t > 0; \quad y_1(t) = t^2$
24. $t^2y'' + 2ty' - 2y = 0, \quad t > 0; \quad y_1(t) = t$
25. $t^2y'' + 3ty' + y = 0, \quad t > 0; \quad y_1(t) = t^{-1}$
26. $t^2y'' - t(t + 2)y' + (t + 2)y = 0, \quad t > 0; \quad y_1(t) = t$
27. $xy'' - y' + 4x^3y = 0, \quad x > 0; \quad y_1(x) = \sin x^2$
28. $(x - 1)y'' - xy' + y = 0, \quad x > 1; \quad y_1(x) = e^x$
29. $x^2y'' - (x - 0.1875)y = 0, \quad x > 0; \quad y_1(x) = x^{1/4}e^{2\sqrt{x}}$
30. $x^2y'' + xy' + (x^2 - 0.25)y = 0, \quad x > 0; \quad y_1(x) = x^{-1/2}\sin x$