In each of Problems 1 through 14, find the general solution of the given differential equation.

1. $y'' - 2y' - 3y = 3e^{2t}$
2. $y'' + 2y' + 5y = 3\sin 2t$
3. $y'' - y' - 2y = -2t + 4t^2$
4. $y'' + y' - 6y = 12e^{3t} + 12e^{-2t}$
5. $y'' - 2y' - 3y = -3te^{-t}$
6. $y'' + 2y' = 3 + 4\sin 2t$
7. $y'' + 9y = t^2e^{3t} + 6$
8. $y'' + 2y' + y = 2e^{-t}$
9. $2y'' + 3y' + y = t^2 + 3\sin t$
10. $y'' + y = 3\sin 2t + t\cos 2t$
11. $u'' + \omega_0^2 u = \cos\omega t, \quad \omega^2 \neq \omega_0^2$
12. $u'' + \omega_0^2 u = \cos\omega_0 t$
13. $y'' + y' + 4y = 2\sinh t$
    Hint: $\sinh t = (e^t - e^{-t})/2$
14. $y'' - y' - 2y = \cosh 2t$
    Hint: $\cosh t = (e^t + e^{-t})/2$

In each of Problems 15 through 20, find the solution of the given initial value problem.

15. $y'' + y' - 2y = 2t, \quad y(0) = 0, \quad y'(0) = 1$
16. $y'' + 4y = t^2 + 3e^t, \quad y(0) = 0, \quad y'(0) = 2$
17. $y'' - 2y' + y = te^t + 4, \quad y(0) = 1, \quad y'(0) = 1$
18. $y'' - 2y' - 3y = 3te^{2t}, \quad y(0) = 1, \quad y'(0) = 0$
19. $y'' + 4y = 3\sin 2t, \quad y(0) = 2, \quad y'(0) = -1$
20. $y'' + 2y' + 5y = 4e^{-t}\cos 2t, \quad y(0) = 1, \quad y'(0) = 0$

In each of Problems 1 through 4, use the method of variation of parameters to find a particular solution of the given differential equation. Then check your answer by using the method of undetermined coefficients.

1. $y'' - 5y' + 6y = 2e^t$
2. $y'' - y' - 2y = 2e^{-t}$
3. $y'' + 2y' + y = 3e^{-t}$
4. $4y'' - 4y' + y = 16e^{t/2}$

In each of Problems 5 through 12, find the general solution of the given differential equation. In Problems 11 and 12, g is an arbitrary continuous function.

5. $y'' + y = \tan t, \quad 0 < t < \pi/2$
6. $y'' + 9y = 9\sec^2 3t, \quad 0 < t < \pi/6$
7. $y'' + 4y' + 4y = t^{-2}e^{-2t}, \quad t > 0$
8. $y'' + 4y = 3\csc 2t, \quad 0 < t < \pi/2$
9. $4y'' + y = 2\sec(t/2), \quad -\pi < t < \pi$
10. $y'' - 2y' + y = e^t/(1 + t^2)$
11. $y'' - 5y' + 6y = g(t)$
12. $y'' + 4y = g(t)$

In each of Problems 13 through 20, verify that the given functions y_1 and y_2 satisfy the corresponding homogeneous equation; then find a particular solution of the given nonhomogeneous equation. In Problems 19 and 20, g is an arbitrary continuous function.

13. $t^2y'' - 2y = 3t^2 - 1, \quad t > 0; \quad y_1(t) = t^2, \quad y_2(t) = t^{-1}$
14. $t^2y'' - t(t+2)y' + (t+2)y = 2t^3, \quad t > 0; \quad y_1(t) = t, \quad y_2(t) = te^t$
15. $ty'' - (1+t)y' + y = t^2e^{2t}, \quad t > 0; \quad y_1(t) = 1 + t, \quad y_2(t) = e^t$
16. $(1-t)y'' + ty' - y = 2(t-1)^2e^{-t}, \quad 0 < t < 1; \quad y_1(t) = e^t, \quad y_2(t) = t$
17. $x^2y'' - 3xy' + 4y = x^2\ln x, \quad x > 0; \quad y_1(x) = x^2, \quad y_2(x) = x^2\ln x$
18. $x^2y'' + xy' + (x^2 - 0.25)y = 3x^{3/2}\sin x, \quad x > 0;$
    $y_1(x) = x^{-1/2}\sin x, \quad y_2(x) = x^{-1/2}\cos x$
19. $(1-x)y'' + xy' - y = g(x), \quad 0 < x < 1; \quad y_1(x) = e^x, \quad y_2(x) = x$
20. $x^2y'' + xy' + (x^2 - 0.25)y = g(x), \quad x > 0; \quad y_1(x) = x^{-1/2}\sin x, \quad y_2(x) = x^{-1/2}\cos x$

In each of Problems 1 through 6, determine intervals in which solutions are sure to exist.

1. $y^{(4)} + 4y''' + 3y = t$
2. $ty''' + (\sin t)y'' + 3y = \cos t$
3. $t(t-1)y^{(4)} + e^t y'' + 4t^2 y = 0$
4. $y''' + ty'' + t^2 y' + t^3 y = \ln t$
5. $(x-1)y^{(4)} + (x+1)y'' + (\tan x)y = 0$
6. $(x^2-4)y^{(6)} + x^2 y''' + 9y = 0$

In each of Problems 7 through 10, determine whether the given functions are linearly dependent or linearly independent. If they are linearly dependent, find a linear relation among them.

7. $f_1(t) = 2t - 3, \quad f_2(t) = t^2 + 1, \quad f_3(t) = 2t^2 - t$
8. $f_1(t) = 2t - 3, \quad f_2(t) = 2t^2 + 1, \quad f_3(t) = 3t^2 + t$
9. $f_1(t) = 2t - 3, \quad f_2(t) = t^2 + 1, \quad f_3(t) = 2t^2 - t, \quad f_4(t) = t^2 + t + 1$
10. $f_1(t) = 2t - 3, \quad f_2(t) = t^3 + 1, \quad f_3(t) = 2t^2 - t, \quad f_4(t) = t^2 + t + 1$

In each of Problems 11 through 16, verify that the given functions are solutions of the differential equation, and determine their Wronskian.

11. $y''' + y' = 0; \quad 1, \quad \cos t, \quad \sin t$
12. $y^{(4)} + y'' = 0; \quad 1, \quad t, \quad \cos t, \quad \sin t$
13. $y''' + 2y'' - y' - 2y = 0; \quad e^t, \quad e^{-t}, \quad e^{-2t}$
14. $y^{(4)} + 2y''' + y'' = 0; \quad 1, \quad t, \quad e^{-t}, \quad te^{-t}$
15. $xy''' - y'' = 0; \quad 1, \quad x, \quad x^3$
16. $x^3 y''' + x^2 y'' - 2xy' + 2y = 0; \quad x, \quad x^2, \quad 1/x$
17. Show that $W(5, \sin^2 t, \cos 2t) = 0$ for all t. Can you establish this result without direct evaluation of the Wronskian?

In each of Problems 7 through 10, follow the procedure illustrated in Example 4 to determine the indicated roots of the given complex number.

7. $1^{1/3}$
8. $(1-i)^{1/2}$
9. $1^{1/4}$
10. $[2(\cos \pi/3 + i \sin \pi/3)]^{1/2}$

In each of Problems 11 through 28, find the general solution of the given differential equation.

11. $y''' - y'' - y' + y = 0$
12. $y''' - 3y'' + 3y' - y = 0$
13. $2y''' - 4y'' - 2y' + 4y = 0$
14. $y^{(4)} - 4y''' + 4y'' = 0$
15. $y^{(6)} + y = 0$
16. $y^{(4)} - 5y'' + 4y = 0$
17. $y^{(6)} - 3y^{(4)} + 3y'' - y = 0$
18. $y^{(6)} - y'' = 0$
19. $y^{(5)} - 3y^{(4)} + 3y''' - 3y'' + 2y' = 0$
20. $y^{(4)} - 8y' = 0$
21. $y^{(8)} + 8y^{(4)} + 16y = 0$
22. $y^{(4)} + 2y'' + y = 0$
23. $y''' - 5y'' + 3y' + y = 0$
24. $y''' + 5y'' + 6y' + 2y = 0$

In each of Problems 1 through 8, determine the general solution of the given differential equation.

1. $y''' - y'' - y' + y = 2e^{-t} + 3$
2. $y^{(4)} - y = 3t + \cos t$
3. $y''' + y'' + y' + y = e^{-t} + 4t$
4. $y''' - y' = 2\sin t$
5. $y^{(4)} - 4y'' = t^2 + e^t$
6. $y^{(4)} + 2y'' + y = 3 + \cos 2t$
7. $y^{(6)} + y''' = t$
8. $y^{(4)} + y''' = \sin 2t$

In each of Problems 9 through 12, find the solution of the given initial value problem. Then plot a graph of the solution.

9. $y''' + 4y' = t;\quad y(0) = y'(0) = 0,\quad y''(0) = 1$
10. $y^{(4)} + 2y'' + y = 3t + 4;\quad y(0) = y'(0) = 0,\quad y''(0) = y'''(0) = 1$
11. $y''' - 3y'' + 2y' = t + e^t;\quad y(0) = 1,\quad y'(0) = -\frac{1}{4},\quad y''(0) = -\frac{3}{2}$
12. $y^{(4)} + 2y''' + y'' + 8y' - 12y = 12\sin t - e^{-t};\quad y(0) = 3,\quad y'(0) = 0,$ $y''(0) = -1,\quad y'''(0) = 2$

In each of Problems 13 through 18, determine a suitable form for $Y(t)$ if the method of undetermined coefficients is to be used. Do not evaluate the constants.

13. $y''' - 2y'' + y' = t^3 + 2e^t$
14. $y''' - y' = te^{-t} + 2\cos t$
15. $y^{(4)} - 2y'' + y = e^t + \sin t$
16. $y^{(4)} + 4y'' = \sin 2t + te^t + 4$
17. $y^{(4)} - y''' - y'' + y' = t^2 + 4 + t\sin t$
18. $y^{(4)} + 2y''' + 2y'' = 3e^t + 2te^{-t} + e^{-t}\sin t$

In each of Problems 1 through 10, find the inverse Laplace transform of the given function.

1. $F(s) = \dfrac{3}{s^2 + 4}$
2. $F(s) = \dfrac{4}{(s-1)^3}$
3. $F(s) = \dfrac{2}{s^2 + 3s - 4}$
4. $F(s) = \dfrac{3s}{s^2 - s - 6}$
5. $F(s) = \dfrac{2s + 2}{s^2 + 2s + 5}$
6. $F(s) = \dfrac{2s - 3}{s^2 - 4}$
7. $F(s) = \dfrac{2s + 1}{s^2 - 2s + 2}$
8. $F(s) = \dfrac{8s^2 - 4s + 12}{s(s^2 + 4)}$
9. $F(s) = \dfrac{1 - 2s}{s^2 + 4s + 5}$
10. $F(s) = \dfrac{2s - 3}{s^2 + 2s + 10}$

In each of Problems 11 through 23, use the Laplace transform to solve the given initial value problem.

11. $y'' - y' - 6y = 0;\quad y(0) = 1,\quad y'(0) = -1$
12. $y'' + 3y' + 2y = 0;\quad y(0) = 1,\quad y'(0) = 0$
13. $y'' - 2y' + 2y = 0;\quad y(0) = 0,\quad y'(0) = 1$
14. $y'' - 4y' + 4y = 0;\quad y(0) = 1,\quad y'(0) = 1$
15. $y'' - 2y' + 4y = 0;\quad y(0) = 2,\quad y'(0) = 0$
16. $y'' + 2y' + 5y = 0;\quad y(0) = 2,\quad y'(0) = -1$
17. $y^{(4)} - 4y''' + 6y'' - 4y' + y = 0;\quad y(0) = 0,\quad y'(0) = 1,\quad y''(0) = 0,\quad y'''(0) = 1$
18. $y^{(4)} - y = 0;\quad y(0) = 1,\quad y'(0) = 0,\quad y''(0) = 1,\quad y'''(0) = 0$
19. $y^{(4)} - 4y = 0;\quad y(0) = 1,\quad y'(0) = 0,\quad y''(0) = -2,\quad y'''(0) = 0$
20. $y'' + \omega^2 y = \cos 2t,\quad \omega^2 \neq 4;\quad y(0) = 1,\quad y'(0) = 0$
21. $y'' - 2y' + 2y = \cos t;\quad y(0) = 1,\quad y'(0) = 0$
22. $y'' - 2y' + 2y = e^{-t};\quad y(0) = 0,\quad y'(0) = 1$
23. $y'' + 2y' + y = 4e^{-t};\quad y(0) = 2,\quad y'(0) = -1$