Section 3.4

2. $exp(2-3i) = e^2 e^{-3i} = e^2(cos\,3 - i\,sin\,3)$.

3. $e^{i\pi} = cos\,\pi + i\,sin\,\pi = -1$.

4. $exp\big(2-\frac{\pi}{2}i\big) = e^2\big(cos\,\frac{\pi}{2} - i\,sin\,\frac{\pi}{2}\big) = -e^2\,i$.

6. $\pi^{-1+2i} = exp[(-1+2i)ln\,\pi] = exp(-ln\,\pi)exp(2\,ln\,\pi\,i) = \frac{1}{\pi}\,exp(2\,ln\,\pi\,i) =$
$= \frac{1}{\pi}[cos\,(2\,ln\,\pi) + i\,sin\,(2\,ln\,\pi)]$.

8. The characteristic equation is $r^2 - 2r + 6 = 0$, with roots $r = 1 \pm i\sqrt{5}$. Hence the general solution is $y = c_1 e^t cos\,\sqrt{5}\,t + c_2\,e^t sin\,\sqrt{5}\,t$.

9. The characteristic equation is $r^2 + 2r - 8 = 0$, with roots $r = -4\,,2$. The roots are *real* and different, hence the general solution is $y = c_1 e^{-4t} + c_2\,e^{2t}$.

10. The characteristic equation is $r^2 + 2r + 2 = 0$, with roots $r = -1 \pm i$. Hence the general solution is $y = c_1 e^{-t} cos\,t + c_2\,e^{-t} sin\,t$.

12. The characteristic equation is $4r^2 + 9 = 0$, with roots $r = \pm\frac{3}{2}\,i$. Hence the general solution is $y = c_1 cos\,\frac{3}{2}t + c_2\,sin\,\frac{3}{2}t$.

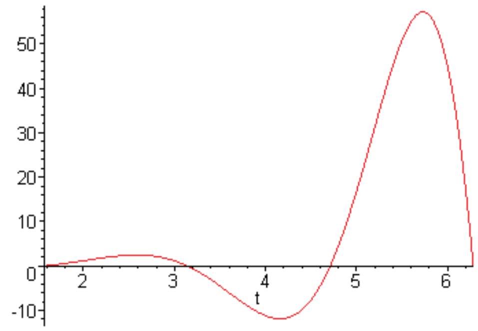
13. The characteristic equation is $r^2 + 2r + 1.25 = 0$, with roots $r = -1 \pm \frac{1}{2}i$. Hence the general solution is $y = c_1 e^{-t} cos\,\frac{1}{2}t + c_2\,e^{-t} sin\,\frac{1}{2}t$.

15. The characteristic equation is $r^2 + r + 1.25 = 0$, with roots $r = -\frac{1}{2} \pm i$. Hence the general solution is $y = c_1 e^{-t/2} cos\,t + c_2\,e^{-t/2} sin\,t$.

16. The characteristic equation is $r^2 + 4r + 6.25 = 0$, with roots $r = -2 \pm \frac{3}{2}\,i$. Hence the general solution is $y = c_1 e^{-2t} cos\,\frac{3}{2}t + c_2\,e^{-2t} sin\,\frac{3}{2}t$.

17. The characteristic equation is $r^2 + 4 = 0$, with roots $r = \pm\,2i$. Hence the general solution is $y = c_1 cos\,2t + c_2\,sin\,2t$. Its derivative is $y' = -2c_1 sin\,2t + 2c_2\,cos\,2t$. Based on the first condition, $y(0) = 0$, we require that $c_1 = 0$. In order to satisfy the condition $y'(0) = 1$, we find that $2c_2 = 1$. The constants are $c_1 = 0$ and $c_2 = 1/2$. Hence the specific solution is $y(t) = \frac{1}{2} sin\,2t$.

19. The characteristic equation is $r^2 - 2r + 5 = 0$, with roots $r = 1 \pm 2i$. Hence the general solution is $y = c_1 e^t cos\,2t + c_2\,e^t sin\,2t$. Based on the condition, $y(\pi/2) = 0$, we require that $c_1 = 0$. It follows that $y = c_2\,e^t sin\,2t$, and so the first derivative is $y' = c_2\,e^t sin\,2t + 2c_2\,e^t cos\,2t$. In order to satisfy the condition $y'(\pi/2) = 2$, we find that $-2e^{\pi/2} c_2 = 2$. Hence we have $c_2 = -e^{-\pi/2}$. Therefore the specific solution is $y(t) = -e^{t-\pi/2}\,sin\,2t$.

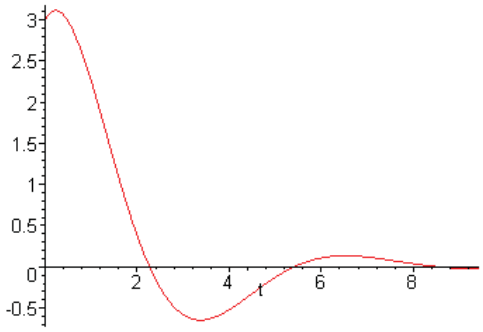


20. The characteristic equation is $r^2 + 1 = 0$, with roots $r = \pm i$. Hence the general solution is $y = c_1 cos\, t + c_2\, sin\, t$. Its derivative is $y' = -c_1 sin\, t + c_2\, cos\, t$. Based on the first condition, $y(\pi/3) = 2$, we require that $c_1 + \sqrt{3}\, c_2 = 4$. In order to satisfy the condition $y'(\pi/3) = -4$, we find that $-\sqrt{3}\, c_1 + c_2 = -8$. Solving these for the constants, $c_1 = 1 + 2\sqrt{3}$ and $c_2 = \sqrt{3} - 2$. Hence the specific solution is a steady oscillation, given by $y(t) = \left(1 + 2\sqrt{3}\right) cos\, t + \left(\sqrt{3} - 2\right) sin\, t$.

21. From Prob. 15, the general solution is $y = c_1 e^{-t/2} cos\, t + c_2\, e^{-t/2} sin\, t$. Invoking the first initial condition, $y(0) = 3$, which implies that $c_1 = 3$. Substituting, it follows that $y = 3e^{-t/2} cos\, t + c_2\, e^{-t/2} sin\, t$, and so the first derivative is

$$y' = -\frac{3}{2} e^{-t/2} cos\, t - 3e^{-t/2} sin\, t + c_2\, e^{-t/2} cos\, t - \frac{c_2}{2} e^{-t/2} sin\, t\,.$$

Invoking the initial condition, $y'(0) = 1$, we find that $-\frac{3}{2} + c_2 = 1$, and so $c_2 = \frac{5}{2}$. Hence the specific solution is $y(t) = 3e^{-t/2} cos\, t + \frac{5}{3}\, e^{-t/2} sin\, t$.



24(a). The characteristic equation is $5r^2 + 2r + 7 = 0$, with roots $r = -\frac{1}{5} \pm i\frac{\sqrt{34}}{5}$. The solution is $u = c_1 e^{-t/5} cos\, \frac{\sqrt{34}}{5} t + c_2 e^{-t/5} sin\, \frac{\sqrt{34}}{5} t$. Invoking the given initial conditions, we obtain the equations for the coefficients: $c_1 = 2\,,\ -2 + \sqrt{34}\, c_2 = 5$. That is, $c_1 = 2\,,\ c_2 = 7/\sqrt{34}$. Hence the specific solution is