note that if the real part of the roots is zero, as in this example, then there is no exponential factor in the solution. Figure 3.3.3 shows the graph of two typical solutions of Eq. (28). In each case the solution is a pure oscillation whose amplitude is determined by the initial conditions. Since there is no exponential factor in the solution (29), the amplitude of each oscillation remains constant in time.

PROBLEMS

In each of Problems 1 through 6, use Euler's formula to write the given expression in the form $a + ib$.

1. $\exp(1 + 2i)$
2. $\exp(2 - 3i)$
3. $e^{i\pi}$
4. $e^{2-(\pi/2)i}$
5. 2^{1-i}
6. π^{-1+2i}

In each of Problems 7 through 16, find the general solution of the given differential equation.

7. $y'' - 2y' + 2y = 0$
8. $y'' - 2y' + 6y = 0$
9. $y'' + 2y' - 8y = 0$
10. $y'' + 2y' + 2y = 0$
11. $y'' + 6y' + 13y = 0$
12. $4y'' + 9y = 0$
13. $y'' + 2y' + 1.25y = 0$
14. $9y'' + 9y' - 4y = 0$
15. $y'' + y' + 1.25y = 0$
16. $y'' + 4y' + 6.25y = 0$

In each of Problems 17 through 22, find the solution of the given initial value problem. Sketch the graph of the solution and describe its behavior for increasing t.

17. $y'' + 4y = 0, \quad y(0) = 0, \quad y'(0) = 1$
18. $y'' + 4y' + 5y = 0, \quad y(0) = 1, \quad y'(0) = 0$
19. $y'' - 2y' + 5y = 0, \quad y(\pi/2) = 0, \quad y'(\pi/2) = 2$
20. $y'' + y = 0, \quad y(\pi/3) = 2, \quad y'(\pi/3) = -4$
21. $y'' + y' + 1.25y = 0, \quad y(0) = 3, \quad y'(0) = 1$
22. $y'' + 2y' + 2y = 0, \quad y(\pi/4) = 2, \quad y'(\pi/4) = -2$
23. Consider the initial value problem

$$3u'' - u' + 2u = 0, \quad u(0) = 2, \quad u'(0) = 0.$$

(a) Find the solution $u(t)$ of this problem.
(b) For $t > 0$, find the first time at which $|u(t)| = 10$.

24. Consider the initial value problem

$$5u'' + 2u' + 7u = 0, \quad u(0) = 2, \quad u'(0) = 1.$$

(a) Find the solution $u(t)$ of this problem.
(b) Find the smallest T such that $|u(t)| \le 0.1$ for all $t > T$.

25. Consider the initial value problem

$$y'' + 2y' + 6y = 0, \quad y(0) = 2, \quad y'(0) = \alpha \ge 0.$$

(a) Find the solution $y(t)$ of this problem.
(b) Find α such that $y = 0$ when $t = 1$.
(c) Find, as a function of α, the smallest positive value of t for which $y = 0$.
(d) Determine the limit of the expression found in part (c) as $\alpha \to \infty$.