3.4 Exercises

1. Verify Theorem 3.11 for the matrix

$$A = \begin{bmatrix} -2 & 3 & 0 \\ 4 & 1 & -3 \\ 2 & 0 & 1 \end{bmatrix}$$

by computing $a_{11}A_{12} + a_{21}A_{22} + a_{31}A_{32}$.

2. Let $A = \begin{bmatrix} 2 & 1 & 3 \\ -1 & 2 & 0 \\ 3 & -2 & 1 \end{bmatrix}$.

(a) Find adj A.

(b) Compute $\det(A)$.

(c) Verify Theorem 3.12; that is, show that

$$A(\operatorname{adj} A) = (\operatorname{adj} A)A = \det(A)I_3.$$

3. Let $A = \begin{bmatrix} 6 & 2 & 8 \\ -3 & 4 & 1 \\ 4 & -4 & 5 \end{bmatrix}$. Follow the directions of Exercise 2.

4. Find the inverse of the matrix in Exercise 2 by the method given in Corollary 3.4.

5. Repeat Exercise 11 of Section 2.3 by the method given in Corollary 3.4. Compare your results with those obtained earlier.

6. Prove that if A is a symmetric matrix, then adj A is symmetric.

7. Use the method given in Corollary 3.4 to find the inverse, if it exists, of

(a) $\begin{bmatrix} 0 & 2 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ -2 & 1 & 5 & 2 \\ 0 & 1 & 0 & 2 \end{bmatrix}$,

(b) $\begin{bmatrix} 4 & 2 & 2 \\ 0 & 1 & 2 \\ 1 & 0 & 3 \end{bmatrix}$, **(c)** $\begin{bmatrix} 3 & 2 \\ -3 & 4 \end{bmatrix}$.

8. Prove that if A is a nonsingular upper triangular matrix, then A^{-1} is upper triangular.

9. Use the method given in Corollary 3.4 to find the inverse of

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \text{if } ad - bc \neq 0.$$

10. Use the method given in Corollary 3.4 to find the inverse of

$$A = \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}.$$

[*Hint*: See Exercise 22 in Section 3.2, where $\det(A)$ is computed.]

11. Use the method given in Corollary 3.4 to find the inverse of

$$A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

12. Use the method given in Corollary 3.4 to find the inverse of

$$A = \begin{bmatrix} 4 & 1 & 2 \\ 0 & -3 & 3 \\ 0 & 0 & 2 \end{bmatrix}.$$

13. Prove that if A is singular, then adj A is singular. [*Hint*: First show that if A is singular, then $A(\operatorname{adj} A) = O$.]

14. Prove that if A is an $n \times n$ matrix, then $\det(\operatorname{adj} A) = [\det(A)]^{n-1}$.

15. Assuming that your software has a command for computing the inverse of a matrix (see Exercise 63 in Section 1.5), read the accompanying software documentation to determine the method used. Is the description closer to that in Section 2.3 or Corollary 3.4? See also the comments in Section 3.6.

3.5 Other Applications of Determinants

We can use the results developed in Theorem 3.12 to obtain another method for solving a linear system of n equations in n unknowns. This method is known as **Cramer's rule**.