and we should choose

$$Y(t) = e^{(\alpha+i\beta)t}(A_0t^n + \cdots + A_n) + e^{(\alpha-i\beta)t}(B_0t^n + \cdots + B_n),$$

or, equivalently,

$$Y(t) = e^{\alpha t}(A_0t^n + \cdots + A_n)\cos\beta t + e^{\alpha t}(B_0t^n + \cdots + B_n)\sin\beta t.$$

Usually, the latter form is preferred. If $\alpha \pm i\beta$ satisfy the characteristic equation corresponding to the homogeneous equation, we must, of course, multiply each of the polynomials by t to increase their degrees by one.

If the nonhomogeneous function involves both $\cos\beta t$ and $\sin\beta t$, it is usually convenient to treat these terms together, since each one individually may give rise to the same form for a particular solution. For example, if $g(t) = t\sin t + 2\cos t$, the form for $Y(t)$ would be

$$Y(t) = (A_0t + A_1)\sin t + (B_0t + B_1)\cos t,$$

provided that $\sin t$ and $\cos t$ are not solutions of the homogeneous equation.

PROBLEMS

In each of Problems 1 through 14, find the general solution of the given differential equation.

1. $y'' - 2y' - 3y = 3e^{2t}$
2. $y'' + 2y' + 5y = 3\sin 2t$
3. $y'' - y' - 2y = -2t + 4t^2$
4. $y'' + y' - 6y = 12e^{3t} + 12e^{-2t}$
5. $y'' - 2y' - 3y = -3te^{-t}$
6. $y'' + 2y' = 3 + 4\sin 2t$
7. $y'' + 9y = t^2e^{3t} + 6$
8. $y'' + 2y' + y = 2e^{-t}$
9. $2y'' + 3y' + y = t^2 + 3\sin t$
10. $y'' + y = 3\sin 2t + t\cos 2t$
11. $u'' + \omega_0^2 u = \cos\omega t, \qquad \omega^2 \neq \omega_0^2$
12. $u'' + \omega_0^2 u = \cos\omega_0 t$
13. $y'' + y' + 4y = 2\sinh t$
    Hint: $\sinh t = (e^t - e^{-t})/2$
14. $y'' - y' - 2y = \cosh 2t$
    Hint: $\cosh t = (e^t + e^{-t})/2$

In each of Problems 15 through 20, find the solution of the given initial value problem.

15. $y'' + y' - 2y = 2t, \qquad y(0) = 0, \quad y'(0) = 1$
16. $y'' + 4y = t^2 + 3e^t, \qquad y(0) = 0, \quad y'(0) = 2$
17. $y'' - 2y' + y = te^t + 4, \qquad y(0) = 1, \quad y'(0) = 1$
18. $y'' - 2y' - 3y = 3te^{2t}, \qquad y(0) = 1, \quad y'(0) = 0$
19. $y'' + 4y = 3\sin 2t, \qquad y(0) = 2, \quad y'(0) = -1$
20. $y'' + 2y' + 5y = 4e^{-t}\cos 2t, \qquad y(0) = 1, \quad y'(0) = 0$

In each of Problems 21 through 28:

(a) Determine a suitable form for $Y(t)$ if the method of undetermined coefficients is to be used.

(b) Use a computer algebra system to find a particular solution of the given equation.

21. $y'' + 3y' = 2t^4 + t^2e^{-3t} + \sin 3t$
22. $y'' + y = t(1 + \sin t)$
23. $y'' - 5y' + 6y = e^t\cos 2t + e^{2t}(3t + 4)\sin t$
24. $y'' + 2y' + 2y = 3e^{-t} + 2e^{-t}\cos t + 4e^{-t}t^2\sin t$
25. $y'' - 4y' + 4y = 2t^2 + 4te^{2t} + t\sin 2t$
26. $y'' + 4y = t^2\sin 2t + (6t + 7)\cos 2t$