```
{a, b}={0, π}; n =10; dx = (b − a)/n;
f = Sin[x]^2;
xvals =Table[N[x], {x, a, b − dx, dx}];
yvals = f /.x → xvals;
boxes = MapThread[Line[{{#1,0},{#1, #3},{#2, #3},{#2, 0}]&,{xvals, xvals + dx, yvals}];
Plot[f, {x, a, b}, Epilog → boxes];
Sum[yvals[[i]] dx, {i, 1, Length[yvals]}]//N
```

Sums of rectangles evaluated at right-hand endpoints can be represented and evaluated by this set of commands.

```
Clear[x, f, a, b, n]
{a, b}={0, π}; n =10; dx = (b − a)/n;
f = Sin[x]^2;
xvals =Table[N[x], {x, a + dx, b, dx}];
yvals = f /.x → xvals;
boxes = MapThread[Line[{{#1,0},{#1, #3},{#2, #3},{#2, 0}]&,{xvals − dx,xvals, yvals}];
Plot[f, {x, a, b}, Epilog → boxes];
Sum[yvals[[i]] dx, {i, 1,Length[yvals]}]//N
```

Sums of rectangles evaluated at midpoints can be represented and evaluated by this set of commands.

```
Clear[x, f, a, b, n]
{a, b}={0, π}; n =10; dx = (b − a)/n;
f = Sin[x]^2;
xvals =Table[N[x], {x, a + dx/2, b − dx/2, dx}];
yvals = f /.x → xvals;
boxes = MapThread[Line[{{#1,0},{#1, #3},{#2, #3},{#2, 0}]&,{xvals − dx/2, xvals + dx/2, yvals}];
Plot[f, {x, a, b},Epilog → boxes];
Sum[yvals[[i]] dx, {i, 1, Length[yvals]}]//N
```

5.4 THE FUNDAMENTAL THEOREM OF CALCULUS

1. $\int_{-2}^{0}(2x+5)\,dx = \left[x^2+5x\right]_{-2}^{0} = (0^2+5(0)) - ((-2)^2+5(-2)) = 6$

2. $\int_{-3}^{4}\left(5-\frac{x}{2}\right)dx = \left[5x-\frac{x^2}{4}\right]_{-3}^{4} = \left(5(4)-\frac{4^2}{4}\right) - \left(5(-3)-\frac{(-3)^2}{4}\right) = \frac{133}{4}$

3. $\int_{0}^{4}\left(3x-\frac{x^3}{4}\right)dx = \left[\frac{3x^2}{2}-\frac{x^4}{16}\right]_{0}^{4} = \left(\frac{3(4)^2}{2}-\frac{4^4}{16}\right) - \left(\frac{3(0)^2}{2}-\frac{(0)^4}{16}\right) = 8$

4. $\int_{-2}^{2}(x^3-2x+3)\,dx = \left[\frac{x^4}{4}-x^2+3x\right]_{-2}^{2} = \left(\frac{2^4}{4}-2^2+3(2)\right) - \left(\frac{(-2)^4}{4}-(-2)^2+3(-2)\right) = 12$

5. $\int_{0}^{1}\left(x^2+\sqrt{x}\right)dx = \left[\frac{x^3}{3}+\frac{2}{3}x^{3/2}\right]_{0}^{1} = \left(\frac{1}{3}+\frac{2}{3}\right) - 0 = 1$

6. $\int_{0}^{5} x^{3/2}\,dx = \left[\frac{2}{5}x^{5/2}\right]_{0}^{5} = \frac{2}{5}(5)^{5/2} - 0 = 2(5)^{3/2} = 10\sqrt{5}$

7. $\int_{1}^{32} x^{-6/5}\,dx = \left[-5x^{-1/5}\right]_{1}^{32} = \left(-\frac{5}{2}\right) - (-5) = \frac{5}{2}$

8. $\int_{-2}^{-1} \frac{2}{x^2}\,dx = \int_{-2}^{-1} 2x^{-2}\,dx = \left[-2x^{-1}\right]_{-2}^{-1} = \left(\frac{-2}{-1}\right) - \left(\frac{-2}{-2}\right) = 1$

9. $\int_0^{\pi} \sin x \, dx = [-\cos x]_0^{\pi} = (-\cos \pi) - (-\cos 0) = -(-1) - (-1) = 2$

10. $\int_0^{\pi} (1 + \cos x) \, dx = [x + \sin x]_0^{\pi} = (\pi + \sin \pi) - (0 + \sin 0) = \pi$

11. $\int_0^{\pi/3} 2 \sec^2 x \, dx = [2 \tan x]_0^{\pi/3} = \left(2 \tan\left(\frac{\pi}{3}\right)\right) - (2 \tan 0) = 2\sqrt{3} - 0 = 2\sqrt{3}$

12. $\int_{\pi/6}^{5\pi/6} \csc^2 x \, dx = [-\cot x]_{\pi/6}^{5\pi/6} = \left(-\cot\left(\frac{5\pi}{6}\right)\right) - \left(-\cot\left(\frac{\pi}{6}\right)\right) = -\left(-\sqrt{3}\right) - \left(-\sqrt{3}\right) = 2\sqrt{3}$

13. $\int_{\pi/4}^{3\pi/4} \csc \theta \cot \theta \, d\theta = [-\csc \theta]_{\pi/4}^{3\pi/4} = \left(-\csc\left(\frac{3\pi}{4}\right)\right) - \left(-\csc\left(\frac{\pi}{4}\right)\right) = -\sqrt{2} - \left(-\sqrt{2}\right) = 0$

14. $\int_0^{\pi/3} 4 \sec u \tan u \, du = [4 \sec u]_0^{\pi/3} = 4 \sec\left(\frac{\pi}{3}\right) - 4 \sec 0 = 4(2) - 4(1) = 4$

15. $\int_{\pi/2}^{0} \frac{1 + \cos 2t}{2} \, dt = \int_{\pi/2}^{0} \left(\frac{1}{2} + \frac{1}{2} \cos 2t\right) dt = \left[\frac{1}{2} t + \frac{1}{4} \sin 2t\right]_{\pi/2}^{0} = \left(\frac{1}{2}(0) + \frac{1}{4} \sin 2(0)\right) - \left(\frac{1}{2}\left(\frac{\pi}{2}\right) + \frac{1}{4} \sin 2\left(\frac{\pi}{2}\right)\right)$
$= -\frac{\pi}{4}$

16. $\int_{-\pi/3}^{\pi/3} \frac{1 - \cos 2t}{2} \, dt = \int_{-\pi/3}^{\pi/3} \left(\frac{1}{2} - \frac{1}{2} \cos 2t\right) dt = \left[\frac{1}{2} t - \frac{1}{4} \sin 2t\right]_{-\pi/3}^{\pi/3}$
$= \left(\frac{1}{2}\left(\frac{\pi}{3}\right) - \frac{1}{4} \sin 2\left(\frac{\pi}{3}\right)\right) - \left(\frac{1}{2}\left(-\frac{\pi}{3}\right) - \frac{1}{4} \sin 2\left(-\frac{\pi}{3}\right)\right) = \frac{\pi}{6} - \frac{1}{4} \sin \frac{2\pi}{3} + \frac{\pi}{6} + \frac{1}{4} \sin\left(\frac{-2\pi}{3}\right) = \frac{\pi}{3} - \frac{\sqrt{3}}{4}$

17. $\int_{-\pi/2}^{\pi/2} (8y^2 + \sin y) \, dy = \left[\frac{8y^3}{3} - \cos y\right]_{-\pi/2}^{\pi/2} = \left(\frac{8\left(\frac{\pi}{2}\right)^3}{3} - \cos \frac{\pi}{2}\right) - \left(\frac{8\left(-\frac{\pi}{2}\right)^3}{3} - \cos\left(-\frac{\pi}{2}\right)\right) = \frac{2\pi^3}{3}$

18. $\int_{-\pi/3}^{-\pi/4} \left(4 \sec^2 t + \frac{\pi}{t^2}\right) dt = \int_{-\pi/3}^{-\pi/4} (4 \sec^2 t + \pi t^{-2}) \, dt = \left[4 \tan t - \frac{\pi}{t}\right]_{-\pi/3}^{-\pi/4}$
$= \left(4 \tan\left(-\frac{\pi}{4}\right) - \frac{\pi}{\left(-\frac{\pi}{4}\right)}\right) - \left(4 \tan\left(\frac{\pi}{3}\right) - \frac{\pi}{\left(-\frac{\pi}{3}\right)}\right) = (4(-1) + 4) - \left(4\left(-\sqrt{3}\right) + 3\right) = 4\sqrt{3} - 3$

19. $\int_1^{-1} (r + 1)^2 \, dr = \int_1^{-1} (r^2 + 2r + 1) \, dr = \left[\frac{r^3}{3} + r^2 + r\right]_1^{-1} = \left(\frac{(-1)^3}{3} + (-1)^2 + (-1)\right) - \left(\frac{1^3}{3} + 1^2 + 1\right) = -\frac{8}{3}$

20. $\int_{-\sqrt{3}}^{\sqrt{3}} (t + 1)(t^2 + 4) \, dt = \int_{-\sqrt{3}}^{\sqrt{3}} (t^3 + t^2 + 4t + 4) \, dt = \left[\frac{t^4}{4} + \frac{t^3}{3} + 2t^2 + 4t\right]_{-\sqrt{3}}^{\sqrt{3}}$
$= \left(\frac{\left(\sqrt{3}\right)^4}{4} + \frac{\left(\sqrt{3}\right)^3}{3} + 2\left(\sqrt{3}\right)^2 + 4\sqrt{3}\right) - \left(\frac{\left(-\sqrt{3}\right)^4}{4} + \frac{\left(-\sqrt{3}\right)^3}{3} + 2\left(-\sqrt{3}\right)^2 + 4\left(-\sqrt{3}\right)\right) = 10\sqrt{3}$

21. $\int_{\sqrt{2}}^{1} \left(\frac{u^7}{2} - \frac{1}{u^5}\right) du = \int_{\sqrt{2}}^{1} \left(\frac{u^7}{2} - u^{-5}\right) du = \left[\frac{u^8}{16} + \frac{1}{4u^4}\right]_{\sqrt{2}}^{1} = \left(\frac{1^8}{16} + \frac{1}{4(1)^4}\right) - \left(\frac{\left(\sqrt{2}\right)^8}{16} + \frac{1}{4\left(\sqrt{2}\right)^4}\right) = -\frac{3}{4}$

22. $\int_{1/2}^{1} \left(\frac{1}{v^3} - \frac{1}{v^4}\right) dv = \int_{1/2}^{1} (v^{-3} - v^{-4}) \, dv = \left[\frac{-1}{2v^2} + \frac{1}{3v^3}\right]_{1/2}^{1} = \left(\frac{-1}{2(1)^2} + \frac{1}{3(1)^3}\right) - \left(\frac{-1}{2\left(\frac{1}{2}\right)^2} + \frac{1}{3\left(\frac{1}{2}\right)^3}\right) = -\frac{5}{6}$

23. $\int_1^{\sqrt{2}} \frac{s^2 + \sqrt{s}}{s^2} \, ds = \int_1^{\sqrt{2}} \left(1 + s^{-3/2}\right) ds = \left[s - \frac{2}{\sqrt{s}}\right]_1^{\sqrt{2}} = \left(\sqrt{2} - \frac{2}{\sqrt{\sqrt{2}}}\right) - \left(1 - \frac{2}{\sqrt{1}}\right) = \sqrt{2} - 2^{3/4} + 1$
$= \sqrt{2} - \sqrt[4]{8} + 1$

24. $\int_9^4 \frac{1-\sqrt{u}}{\sqrt{u}}\,du = \int_9^4 \left(u^{-1/2}-1\right)du = \left[2\sqrt{u}-u\right]_9^4 = \left(2\sqrt{4}-4\right)-\left(2\sqrt{9}-9\right) = 3$

25. $\int_{-4}^4 |x|\,dx = \int_{-4}^0 |x|\,dx + \int_0^4 |x|\,dx = -\int_{-4}^0 x\,dx + \int_0^4 x\,dx = \left[-\frac{x^2}{2}\right]_{-4}^0 + \left[\frac{x^2}{2}\right]_0^4 = \left(-\frac{0^2}{2}+\frac{(-4)^2}{2}\right)+\left(\frac{4^2}{2}-\frac{0^2}{2}\right)$
$= 16$

26. $\int_0^\pi \frac{1}{2}(\cos x + |\cos x|)\,dx = \int_0^{\pi/2} \frac{1}{2}(\cos x+\cos x)\,dx + \int_{\pi/2}^\pi \frac{1}{2}(\cos x-\cos x)\,dx = \int_0^{\pi/2}\cos x\,dx = [\sin x]_0^{\pi/2}$
$= \sin\frac{\pi}{2} - \sin 0 = 1$

27. (a) $\int_0^{\sqrt{x}} \cos t\,dt = [\sin t]_0^{\sqrt{x}} = \sin\sqrt{x} - \sin 0 = \sin\sqrt{x} \Rightarrow \frac{d}{dx}\left(\int_0^{\sqrt{x}}\cos t\,dt\right) = \frac{d}{dx}\left(\sin\sqrt{x}\right) = \cos\sqrt{x}\left(\frac{1}{2}x^{-1/2}\right)$
$= \frac{\cos\sqrt{x}}{2\sqrt{x}}$

(b) $\frac{d}{dx}\left(\int_0^{\sqrt{x}}\cos t\,dt\right) = \left(\cos\sqrt{x}\right)\left(\frac{d}{dx}\left(\sqrt{x}\right)\right) = \left(\cos\sqrt{x}\right)\left(\frac{1}{2}x^{-1/2}\right) = \frac{\cos\sqrt{x}}{2\sqrt{x}}$

28. (a) $\int_1^{\sin x} 3t^2\,dt = \left[t^3\right]_1^{\sin x} = \sin^3 x - 1 \Rightarrow \frac{d}{dx}\left(\int_1^{\sin x}3t^2\,dt\right) = \frac{d}{dx}\left(\sin^3 x - 1\right) = 3\sin^2 x\cos x$

(b) $\frac{d}{dx}\left(\int_1^{\sin x}3t^2\,dt\right) = \left(3\sin^2 x\right)\left(\frac{d}{dx}(\sin x)\right) = 3\sin^2 x\cos x$

29. (a) $\int_0^{t^4}\sqrt{u}\,du = \int_0^{t^4} u^{1/2}\,du = \left[\frac{2}{3}u^{3/2}\right]_0^{t^4} = \frac{2}{3}\left(t^4\right)^{3/2} - 0 = \frac{2}{3}t^6 \Rightarrow \frac{d}{dt}\left(\int_0^{t^4}\sqrt{u}\,du\right) = \frac{d}{dt}\left(\frac{2}{3}t^6\right) = 4t^5$

(b) $\frac{d}{dt}\left(\int_0^{t^4}\sqrt{u}\,du\right) = \sqrt{t^4}\left(\frac{d}{dt}\left(t^4\right)\right) = t^2\left(4t^3\right) = 4t^5$

30. (a) $\int_0^{\tan\theta}\sec^2 y\,dy = [\tan y]_0^{\tan\theta} = \tan(\tan\theta) - 0 = \tan(\tan\theta) \Rightarrow \frac{d}{d\theta}\left(\int_0^{\tan\theta}\sec^2 y\,dy\right) = \frac{d}{d\theta}(\tan(\tan\theta))$
$= \left(\sec^2(\tan\theta)\right)\sec^2\theta$

(b) $\frac{d}{d\theta}\left(\int_0^{\tan\theta}\sec^2 y\,dy\right) = \left(\sec^2(\tan\theta)\right)\left(\frac{d}{d\theta}(\tan\theta)\right) = \left(\sec^2(\tan\theta)\right)\sec^2\theta$

31. $y = \int_0^x \sqrt{1+t^2}\,dt \Rightarrow \frac{dy}{dx} = \sqrt{1+x^2}$

32. $y = \int_1^x \frac{1}{t}\,dt \Rightarrow \frac{dy}{dx} = \frac{1}{x},\ x > 0$

33. $y = \int_{\sqrt{x}}^0 \sin t^2\,dt = -\int_0^{\sqrt{x}}\sin t^2\,dt \Rightarrow \frac{dy}{dx} = -\left(\sin\left(\sqrt{x}\right)^2\right)\left(\frac{d}{dx}\left(\sqrt{x}\right)\right) = -(\sin x)\left(\frac{1}{2}x^{-1/2}\right) = -\frac{\sin x}{2\sqrt{x}}$

34. $y = \int_0^{x^2}\cos\sqrt{t}\,dt \Rightarrow \frac{dy}{dx} = \left(\cos\sqrt{x^2}\right)\left(\frac{d}{dx}\left(x^2\right)\right) = 2x\cos|x|$

35. $y = \int_0^{\sin x}\frac{dt}{\sqrt{1-t^2}},\ |x| < \frac{\pi}{2} \Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-\sin^2 x}}\left(\frac{d}{dx}(\sin x)\right) = \frac{1}{\sqrt{\cos^2 x}}(\cos x) = \frac{\cos x}{|\cos x|} = \frac{\cos x}{\cos x} = 1$ since $|x| < \frac{\pi}{2}$

36. $y = \int_0^{\tan x}\frac{dt}{1+t^2} \Rightarrow \frac{dy}{dx} = \left(\frac{1}{1+\tan^2 x}\right)\left(\frac{d}{dx}(\tan x)\right) = \left(\frac{1}{\sec^2 x}\right)\left(\sec^2 x\right) = 1$

37. $-x^2 - 2x = 0 \Rightarrow -x(x+2) = 0 \Rightarrow x = 0$ or $x = -2$; Area

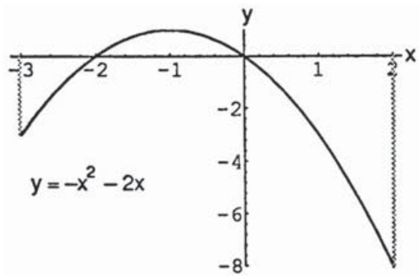
$= -\int_{-3}^{-2} (-x^2 - 2x)dx + \int_{-2}^{0} (-x^2 - 2x)dx - \int_{0}^{2} (-x^2 - 2x)dx$

$= -\left[-\frac{x^3}{3} - x^2\right]_{-3}^{-2} + \left[-\frac{x^3}{3} - x^2\right]_{-2}^{0} - \left[-\frac{x^3}{3} - x^2\right]_{0}^{2}$

$= -\left(\left(-\frac{(-2)^3}{3} - (-2)^2\right) - \left(-\frac{(-3)^3}{3} - (-3)^2\right)\right)$

$+ \left(\left(-\frac{0^3}{3} - 0^2\right) - \left(-\frac{(-2)^3}{3} - (-2)^2\right)\right)$

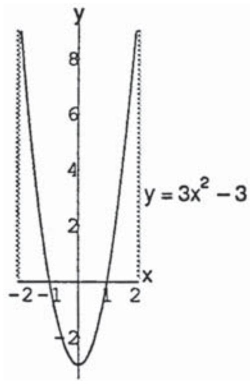
$- \left(\left(-\frac{2^3}{3} - 2^2\right) - \left(-\frac{0^3}{3} - 0^2\right)\right) = \frac{28}{3}$



38. $3x^2 - 3 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$; because of symmetry about the y-axis, Area $= 2\left(-\int_{0}^{1} (3x^2 - 3)dx + \int_{1}^{2} (3x^2 - 3)dx\right)$

$2\left(-[x^3 - 3x]_0^1 + [x^3 - 3x]_1^2\right) = 2\left[-\left((1^3 - 3(1)) - (0^3 - 3(0))\right)\right.$

$\left. + \left((2^3 - 3(2)) - (1^3 - 3(1))\right)\right] = 2(6) = 12$



39. $x^3 - 3x^2 + 2x = 0 \Rightarrow x(x^2 - 3x + 2) = 0$

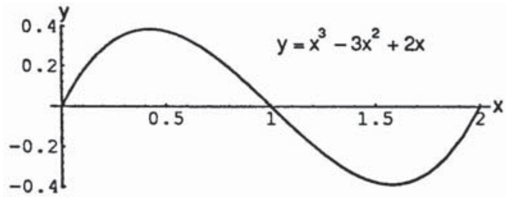
$\Rightarrow x(x-2)(x-1) = 0 \Rightarrow x = 0, 1,$ or 2;

Area $= \int_{0}^{1} (x^3 - 3x^2 + 2x)dx - \int_{1}^{2} (x^3 - 3x^2 + 2x)dx$

$= \left[\frac{x^4}{4} - x^3 + x^2\right]_0^1 - \left[\frac{x^4}{4} - x^3 + x^2\right]_1^2$

$= \left(\frac{1^4}{4} - 1^3 + 1^2\right) - \left(\frac{0^4}{4} - 0^3 + 0^2\right)$

$- \left[\left(\frac{2^4}{4} - 2^3 + 2^2\right) - \left(\frac{1^4}{4} - 1^3 + 1^2\right)\right] = \frac{1}{2}$

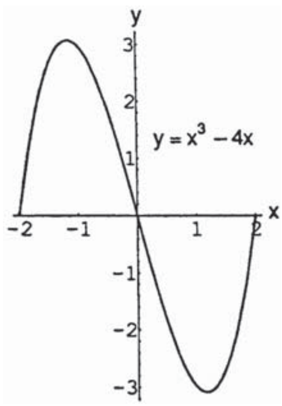


40. $x^3 - 4x = 0 \Rightarrow x(x^2 - 4) = 0 \Rightarrow x(x-2)(x+2) = 0$

$\Rightarrow x = 0, 2,$ or -2. Area $= \int_{-2}^{0} (x^3 - 4x)dx - \int_{0}^{2} (x^3 - 4x)dx$

$= \left[\frac{x^4}{4} - 2x^2\right]_{-2}^{0} - \left[\frac{x^4}{4} - 2x^2\right]_{0}^{2} = \left(\frac{0^4}{4} - 2(0)^2\right)$

$- \left(\frac{(-2)^4}{4} - 2(-2)^2\right) - \left[\left(\frac{2^4}{4} - 2(2)^2\right) - \left(\frac{0^4}{4} - 2(0)^2\right)\right] = 8$

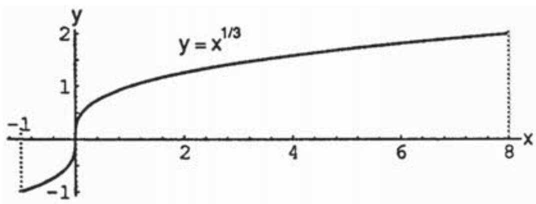


41. $x^{1/3} = 0 \Rightarrow x = 0$; Area $= -\int_{-1}^{0} x^{1/3}\,dx + \int_{0}^{8} x^{1/3}\,dx$

$= \left[-\frac{3}{4}x^{4/3}\right]_{-1}^{0} + \left[\frac{3}{4}x^{4/3}\right]_{0}^{8}$

$= \left(-\frac{3}{4}(0)^{4/3}\right) - \left(-\frac{3}{4}(-1)^{4/3}\right) + \left(\frac{3}{4}(8)^{4/3}\right) - \left(\frac{3}{4}(0)^{4/3}\right)$

$= \frac{51}{4}$



42. $x^{1/3} - x = 0 \Rightarrow x^{1/3}\left(1 - x^{2/3}\right) = 0 \Rightarrow x^{1/3} = 0$ or $1 - x^{2/3} = 0 \Rightarrow x = 0$ or $1 = x^{2/3} \Rightarrow x = 0$ or $1 = x^2 \Rightarrow x = 0$ or ± 1;

$\text{Area} = -\int_{-1}^{0}\left(x^{1/3} - x\right)dx + \int_{0}^{1}\left(x^{1/3} - x\right)dx - \int_{1}^{8}\left(x^{1/3} - x\right)dx$

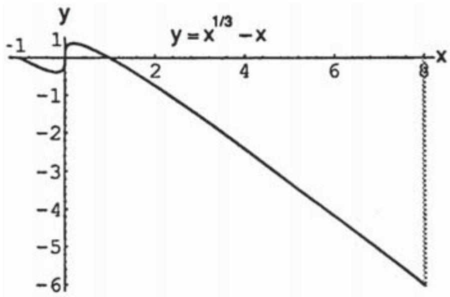
$= -\left[\frac{3}{4}x^{4/3} - \frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{3}{4}x^{4/3} - \frac{x^2}{2}\right]_{0}^{1} - \left[\frac{3}{4}x^{4/3} - \frac{x^2}{2}\right]_{1}^{8}$

$= -\left[\left(\frac{3}{4}(0)^{4/3} - \frac{0^2}{2}\right) - \left(\frac{3}{4}(-1)^{4/3} - \frac{(-1)^2}{2}\right)\right]$

$+ \left[\left(\frac{3}{4}(1)^{4/3} - \frac{1^2}{2}\right) - \left(\frac{3}{4}(0)^{4/3} - \frac{0^2}{2}\right)\right]$

$- \left[\left(\frac{3}{4}(8)^{4/3} - \frac{8^2}{2}\right) - \left(\frac{3}{4}(1)^{4/3} - \frac{1^2}{2}\right)\right]$

$= \frac{1}{4} + \frac{1}{4} - \left(-20 - \frac{3}{4} + \frac{1}{2}\right) = \frac{83}{4}$



43. The area of the rectangle bounded by the lines $y = 2$, $y = 0$, $x = \pi$, and $x = 0$ is 2π. The area under the curve $y = 1 + \cos x$ on $[0, \pi]$ is $\int_{0}^{\pi}(1 + \cos x)\,dx = [x + \sin x]_0^{\pi} = (\pi + \sin \pi) - (0 + \sin 0) = \pi$. Therefore the area of the shaded region is $2\pi - \pi = \pi$.

44. The area of the rectangle bounded by the lines $x = \frac{\pi}{6}$, $x = \frac{5\pi}{6}$, $y = \sin\frac{\pi}{6} = \frac{1}{2} = \sin\frac{5\pi}{6}$, and $y = 0$ is $\frac{1}{2}\left(\frac{5\pi}{6} - \frac{\pi}{6}\right) = \frac{\pi}{3}$. The area under the curve $y = \sin x$ on $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$ is $\int_{\pi/6}^{5\pi/6}\sin x\,dx = [-\cos x]_{\pi/6}^{5\pi/6}$ $= \left(-\cos\frac{5\pi}{6}\right) - \left(-\cos\frac{\pi}{6}\right) = -\left(-\frac{\sqrt{3}}{2}\right) + \frac{\sqrt{3}}{2} = \sqrt{3}$. Therefore the area of the shaded region is $\sqrt{3} - \frac{\pi}{3}$.

45. On $\left[-\frac{\pi}{4}, 0\right]$: The area of the rectangle bounded by the lines $y = \sqrt{2}$, $y = 0$, $\theta = 0$, and $\theta = -\frac{\pi}{4}$ is $\sqrt{2}\left(\frac{\pi}{4}\right)$ $= \frac{\pi\sqrt{2}}{4}$. The area between the curve $y = \sec\theta\tan\theta$ and $y = 0$ is $-\int_{-\pi/4}^{0}\sec\theta\tan\theta\,d\theta = [-\sec\theta]_{-\pi/4}^{0}$ $= (-\sec 0) - \left(-\sec\left(-\frac{\pi}{4}\right)\right) = \sqrt{2} - 1$. Therefore the area of the shaded region on $\left[-\frac{\pi}{4}, 0\right]$ is $\frac{\pi\sqrt{2}}{4} + \left(\sqrt{2} - 1\right)$. On $\left[0, \frac{\pi}{4}\right]$: The area of the rectangle bounded by $\theta = \frac{\pi}{4}$, $\theta = 0$, $y = \sqrt{2}$, and $y = 0$ is $\sqrt{2}\left(\frac{\pi}{4}\right) = \frac{\pi\sqrt{2}}{4}$. The area under the curve $y = \sec\theta\tan\theta$ is $\int_{0}^{\pi/4}\sec\theta\tan\theta\,d\theta = [\sec\theta]_0^{\pi/4} = \sec\frac{\pi}{4} - \sec 0 = \sqrt{2} - 1$. Therefore the area of the shaded region on $\left[0, \frac{\pi}{4}\right]$ is $\frac{\pi\sqrt{2}}{4} - \left(\sqrt{2} - 1\right)$. Thus, the area of the total shaded region is $\left(\frac{\pi\sqrt{2}}{4} + \sqrt{2} - 1\right) + \left(\frac{\pi\sqrt{2}}{4} - \sqrt{2} + 1\right) = \frac{\pi\sqrt{2}}{2}$.

46. The area of the rectangle bounded by the lines $y = 2$, $y = 0$, $t = -\frac{\pi}{4}$, and $t = 1$ is $2\left(1 - \left(-\frac{\pi}{4}\right)\right) = 2 + \frac{\pi}{2}$. The area under the curve $y = \sec^2 t$ on $\left[-\frac{\pi}{4}, 0\right]$ is $\int_{-\pi/4}^{0}\sec^2 t\,dt = [\tan t]_{-\pi/4}^{0} = \tan 0 - \tan\left(-\frac{\pi}{4}\right) = 1$. The area under the curve $y = 1 - t^2$ on $[0, 1]$ is $\int_{0}^{1}(1 - t^2)\,dt = \left[t - \frac{t^3}{3}\right]_0^1 = \left(1 - \frac{1^3}{3}\right) - \left(0 - \frac{0^3}{3}\right) = \frac{2}{3}$. Thus, the total area under the curves on $\left[-\frac{\pi}{4}, 1\right]$ is $1 + \frac{2}{3} = \frac{5}{3}$. Therefore the area of the shaded region is $\left(2 + \frac{\pi}{2}\right) - \frac{5}{3} = \frac{1}{3} + \frac{\pi}{2}$.

47. $y = \int_{\pi}^{x}\frac{1}{t}\,dt - 3 \Rightarrow \frac{dy}{dx} = \frac{1}{x}$ and $y(\pi) = \int_{\pi}^{\pi}\frac{1}{t}\,dt - 3 = 0 - 3 = -3 \Rightarrow$ (d) is a solution to this problem.

48. $y = \int_{-1}^{x}\sec t\,dt + 4 \Rightarrow \frac{dy}{dx} = \sec x$ and $y(-1) = \int_{-1}^{-1}\sec t\,dt + 4 = 0 + 4 = 4 \Rightarrow$ (c) is a solution to this problem.

49. $y = \int_{0}^{x}\sec t\,dt + 4 \Rightarrow \frac{dy}{dx} = \sec x$ and $y(0) = \int_{0}^{0}\sec t\,dt + 4 = 0 + 4 = 4 \Rightarrow$ (b) is a solution to this problem.

50. $y = \int_1^x \frac{1}{t}\,dt - 3 \Rightarrow \frac{dy}{dx} = \frac{1}{x}$ and $y(1) = \int_1^1 \frac{1}{t}\,dt - 3 = 0 - 3 = -3 \Rightarrow$ (a) is a solution to this problem.

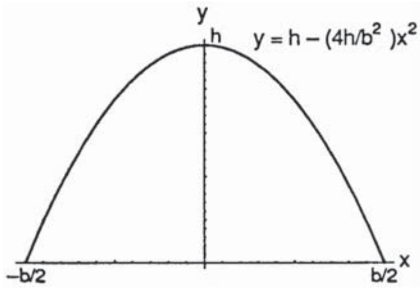
51. $y = \int_2^x \sec t\,dt + 3$

52. $y = \int_1^x \sqrt{1+t^2}\,dt - 2$

53. $s = \int_{t_0}^t f(x)\,dx + s_0$

54. $v = \int_{t_0}^t g(x)\,dx + v_0$

55. Area $= \int_{-b/2}^{b/2} \left(h - \left(\frac{4h}{b^2}\right)x^2\right) dx = \left[hx - \frac{4hx^3}{3b^2}\right]_{-b/2}^{b/2}$
$= \left(h\left(\frac{b}{2}\right) - \frac{4h\left(\frac{b}{2}\right)^3}{3b^2}\right) - \left(h\left(-\frac{b}{2}\right) - \frac{4h\left(-\frac{b}{2}\right)^3}{3b^2}\right)$
$= \left(\frac{bh}{2} - \frac{bh}{6}\right) - \left(-\frac{bh}{2} + \frac{bh}{6}\right) = bh - \frac{bh}{3} = \frac{2}{3}bh$



56. $r = \int_0^3 \left(2 - \frac{2}{(x+1)^2}\right) dx = 2\int_0^3 \left(1 - \frac{1}{(x+1)^2}\right) dx = 2\left[x - \left(\frac{-1}{x+1}\right)\right]_0^3 = 2\left[\left(3 + \frac{1}{(3+1)}\right) - \left(0 + \frac{1}{(0+1)}\right)\right]$
$= 2\left[3\frac{1}{4} - 1\right] = 2\left(2\frac{1}{4}\right) = 4.5$ or \$4500

57. $\frac{dc}{dx} = \frac{1}{2\sqrt{x}} = \frac{1}{2}x^{-1/2} \Rightarrow c = \int_0^x \frac{1}{2}t^{-1/2}dt = \left[t^{1/2}\right]_0^x = \sqrt{x}$
$c(100) - c(1) = \sqrt{100} - \sqrt{1} = \$9.00$

58. By Exercise 57, $c(400) - c(100) = \sqrt{400} - \sqrt{100} = 20 - 10 = \$10.00$

59. (a) $v = \frac{ds}{dt} = \frac{d}{dt}\int_0^t f(x)\,dx = f(t) \Rightarrow v(5) = f(5) = 2$ m/sec
(b) $a = \frac{df}{dt}$ is negative since the slope of the tangent line at $t = 5$ is negative
(c) $s = \int_0^3 f(x)\,dx = \frac{1}{2}(3)(3) = \frac{9}{2}$ m since the integral is the area of the triangle formed by $y = f(x)$, the x-axis, and $x = 3$
(d) $t = 6$ since from $t = 6$ to $t = 9$, the region lies below the x-axis
(e) At $t = 4$ and $t = 7$, since there are horizontal tangents there
(f) Toward the origin between $t = 6$ and $t = 9$ since the velocity is negative on this interval. Away from the origin between $t = 0$ and $t = 6$ since the velocity is positive there.
(g) Right or positive side, because the integral of f from 0 to 9 is positive, there being more area above the x-axis than below it.

60. (a) $v = \frac{dg}{dt} = \frac{d}{dt}\int_0^t g(x)\,dx = g(t) \Rightarrow v(3) = g(3) = 0$ m/sec.
(b) $a = \frac{df}{dt}$ is positive, since the slope of the tangent line at $t = 3$ is positive
(c) At $t = 3$, the particle's position is $\int_0^3 g(x)\,dx = \frac{1}{2}(3)(-6) = -9$
(d) The particle passes through the origin at $t = 6$ because $s(6) = \int_0^6 g(x)\,dx = 0$
(e) At $t = 7$, since there is a horizontal tangent there
(f) The particle starts at the origin and moves away to the left for $0 < t < 3$. It moves back toward the origin for $3 < t < 6$, passes through the origin at $t = 6$, and moves away to the right for $t > 6$.
(g) Right side, since its position at $t = 9$ is positive, there being more area above the x-axis than below it at $t = 9$.