EXERCISES 5.4

Evaluating Integrals

Evaluate the integrals in Exercises 1–26.

1. $\int_{-2}^{0}(2x+5)\,dx$
2. $\int_{-3}^{4}\left(5-\frac{x}{2}\right)dx$
3. $\int_{0}^{4}\left(3x-\frac{x^3}{4}\right)dx$
4. $\int_{-2}^{2}(x^3-2x+3)\,dx$
5. $\int_{0}^{1}\left(x^2+\sqrt{x}\right)dx$
6. $\int_{0}^{5}x^{3/2}\,dx$
7. $\int_{1}^{32}x^{-6/5}\,dx$
8. $\int_{-2}^{-1}\frac{2}{x^2}\,dx$
9. $\int_{0}^{\pi}\sin x\,dx$
10. $\int_{0}^{\pi}(1+\cos x)\,dx$
11. $\int_{0}^{\pi/3}2\sec^2 x\,dx$
12. $\int_{\pi/6}^{5\pi/6}\csc^2 x\,dx$
13. $\int_{\pi/4}^{3\pi/4}\csc\theta\cot\theta\,d\theta$
14. $\int_{0}^{\pi/3}4\sec u\tan u\,du$
15. $\int_{\pi/2}^{0}\frac{1+\cos 2t}{2}\,dt$
16. $\int_{-\pi/3}^{\pi/3}\frac{1-\cos 2t}{2}\,dt$
17. $\int_{-\pi/2}^{\pi/2}(8y^2+\sin y)\,dy$
18. $\int_{-\pi/3}^{-\pi/4}\left(4\sec^2 t+\frac{\pi}{t^2}\right)dt$
19. $\int_{1}^{-1}(r+1)^2\,dr$
20. $\int_{-\sqrt{3}}^{\sqrt{3}}(t+1)(t^2+4)\,dt$
21. $\int_{\sqrt{2}}^{1}\left(\frac{u^7}{2}-\frac{1}{u^5}\right)du$
22. $\int_{1/2}^{1}\left(\frac{1}{v^3}-\frac{1}{v^4}\right)dv$
23. $\int_{1}^{\sqrt{2}}\frac{s^2+\sqrt{s}}{s^2}\,ds$
24. $\int_{9}^{4}\frac{1-\sqrt{u}}{\sqrt{u}}\,du$
25. $\int_{-4}^{4}|x|\,dx$
26. $\int_{0}^{\pi}\frac{1}{2}(\cos x+|\cos x|)\,dx$

Derivatives of Integrals

Find the derivatives in Exercises 27–30

a. by evaluating the integral and differentiating the result.

b. by differentiating the integral directly.

27. $\frac{d}{dx}\int_{0}^{\sqrt{x}}\cos t\,dt$
28. $\frac{d}{dx}\int_{1}^{\sin x}3t^2\,dt$
29. $\frac{d}{dt}\int_{0}^{t^4}\sqrt{u}\,du$
30. $\frac{d}{d\theta}\int_{0}^{\tan\theta}\sec^2 y\,dy$

Find dy/dx in Exercises 31–36.

31. $y=\int_{0}^{x}\sqrt{1+t^2}\,dt$
32. $y=\int_{1}^{x}\frac{1}{t}\,dt,\quad x>0$
33. $y=\int_{\sqrt{x}}^{0}\sin(t^2)\,dt$
34. $y=\int_{0}^{x^2}\cos\sqrt{t}\,dt$
35. $y=\int_{0}^{\sin x}\frac{dt}{\sqrt{1-t^2}},\quad |x|<\frac{\pi}{2}$
36. $y=\int_{\tan x}^{0}\frac{dt}{1+t^2}$

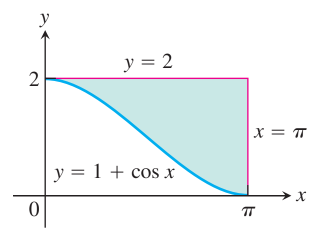
Area

In Exercises 37–42, find the total area between the region and the x-axis.

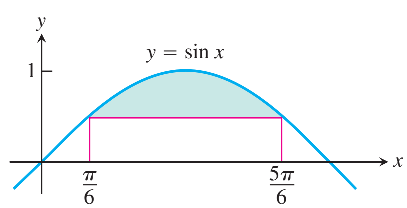
37. $y=-x^2-2x,\quad -3\le x\le 2$
38. $y=3x^2-3,\quad -2\le x\le 2$
39. $y=x^3-3x^2+2x,\quad 0\le x\le 2$
40. $y=x^3-4x,\quad -2\le x\le 2$
41. $y=x^{1/3},\quad -1\le x\le 8$
42. $y=x^{1/3}-x,\quad -1\le x\le 8$

Find the areas of the shaded regions in Exercises 43–46.

43.



44.



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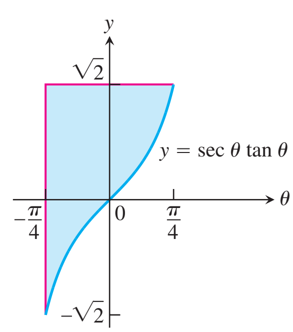

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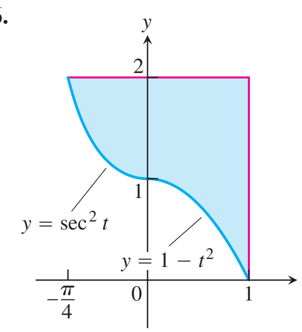


Initial Value Problems

Each of the following functions solves one of the initial value problems in Exercises 47–50. Which function solves which problem? Give brief reasons for your answers.

a. $y = \int_1^x \frac{1}{t}\,dt - 3$ **b.** $y = \int_0^x \sec t\,dt + 4$

c. $y = \int_{-1}^x \sec t\,dt + 4$ **d.** $y = \int_{\pi}^x \frac{1}{t}\,dt - 3$

47. $\frac{dy}{dx} = \frac{1}{x}, \quad y(\pi) = -3$ **48.** $y' = \sec x, \quad y(-1) = 4$

49. $y' = \sec x, \quad y(0) = 4$ **50.** $y' = \frac{1}{x}, \quad y(1) = -3$

Express the solutions of the initial value problems in Exercises 51–54 in terms of integrals.

51. $\frac{dy}{dx} = \sec x, \quad y(2) = 3$

52. $\frac{dy}{dx} = \sqrt{1 + x^2}, \quad y(1) = -2$

53. $\frac{ds}{dt} = f(t), \quad s(t_0) = s_0$

54. $\frac{dv}{dt} = g(t), \quad v(t_0) = v_0$

Applications

55. Archimedes' area formula for parabolas Archimedes (287–212 B.C.), inventor, military engineer, physicist, and the greatest mathematician of classical times in the Western world, discovered that the area under a parabolic arch is two-thirds the base times the height. Sketch the parabolic arch $y = h - (4h/b^2)x^2$, $-b/2 \le x \le b/2$, assuming that h and b are positive. Then use calculus to find the area of the region enclosed between the arch and the x-axis.

56. Revenue from marginal revenue Suppose that a company's marginal revenue from the manufacture and sale of egg beaters is

$$\frac{dr}{dx} = 2 - 2/(x + 1)^2,$$

where r is measured in thousands of dollars and x in thousands of units. How much money should the company expect from a production run of $x = 3$ thousand egg beaters? To find out, integrate the marginal revenue from $x = 0$ to $x = 3$.

57. Cost from marginal cost The marginal cost of printing a poster when x posters have been printed is

$$\frac{dc}{dx} = \frac{1}{2\sqrt{x}}$$

dollars. Find $c(100) - c(1)$, the cost of printing posters 2–100.

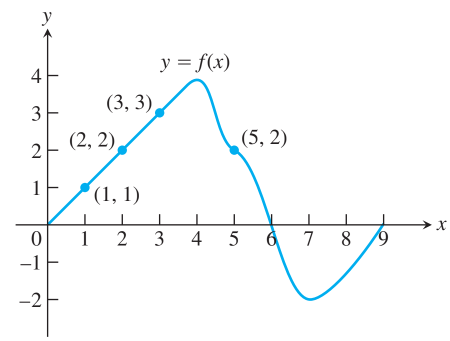
58. (*Continuation of Exercise 57.*) Find $c(400) - c(100)$, the cost of printing posters 101–400.

Drawing Conclusions About Motion from Graphs

59. Suppose that f is the differentiable function shown in the accompanying graph and that the position at time t (sec) of a particle moving along a coordinate axis is

$$s = \int_0^t f(x)\,dx$$

meters. Use the graph to answer the following questions. Give reasons for your answers.



a. What is the particle's velocity at time $t = 5$?

b. Is the acceleration of the particle at time $t = 5$ positive, or negative?

c. What is the particle's position at time $t = 3$?

d. At what time during the first 9 sec does s have its largest value?

e. Approximately when is the acceleration zero?

f. When is the particle moving toward the origin? away from the origin?

g. On which side of the origin does the particle lie at time $t = 9$?

60. Suppose that g is the differentiable function graphed here and that the position at time t (sec) of a particle moving along a coordinate axis is

$$s = \int_0^t g(x)\,dx$$

meters. Use the graph to answer the following questions. Give reasons for your answers.