value(q2);

71-80. Example CAS commands:

Mathematica: (assigned function and values for a, and b may vary)

For transcendental functions the FindRoot is needed instead of the Solve command.

The Map command executes FindRoot over a set of initial guesses

Initial guesses will vary as the functions vary.

```
Clear[x, f, F]
{a, b}= {0, 2π}; f[x_] = Sin[2x] Cos[x/3]
F[x_] = Integrate[f[t], {t, a, x}]
Plot[{f[x], F[x]},{x, a, b}]
x/.Map[FindRoot[F'[x]==0, {x, #}] &,{2, 3, 5, 6}]
x/.Map[FindRoot[f'[x]==0, {x, #}] &,{1, 2, 4, 5, 6}]
```

Slightly alter above commands for 75 - 80.

```
Clear[x, f, F, u]
a=0;  f[x_] = x^2 − 2x − 3
u[x_] = 1 − x^2
F[x_] = Integrate[f[t], {t, a, u(x)}]
x/.Map[FindRoot[F'[x]==0,{x, #}] &,{1, 2, 3, 4}]
x/.Map[FindRoot[F''[x]==0,{x,#}] &,{1, 2, 3, 4}]
```

After determining an appropriate value for b, the following can be entered

```
b = 4;
Plot[{F[x], {x, a, b}]
```

5.5 INDEFINTE INTEGRALS AND THE SUBSTITUTION RULE

1. Let $u = 3x \Rightarrow du = 3\,dx \Rightarrow \frac{1}{3}\,du = dx$

   $\int \sin 3x\,dx = \int \frac{1}{3} \sin u\,du = -\frac{1}{3}\cos u + C = -\frac{1}{3}\cos 3x + C$

2. Let $u = 2x^2 \Rightarrow du = 4x\,dx \Rightarrow \frac{1}{4}\,du = x\,dx$

   $\int x \sin(2x^2)\,dx = \int \frac{1}{4} \sin u\,du = -\frac{1}{4}\cos u + C = -\frac{1}{4}\cos 2x^2 + C$

3. Let $u = 2t \Rightarrow du = 2\,dt \Rightarrow \frac{1}{2}\,du = dt$

   $\int \sec 2t \tan 2t\,dt = \int \frac{1}{2} \sec u \tan u\,du = \frac{1}{2}\sec u + C = \frac{1}{2}\sec 2t + C$

4. Let $u = 1 - \cos\frac{t}{2} \Rightarrow du = \frac{1}{2}\sin\frac{t}{2}\,dt \Rightarrow 2\,du = \sin\frac{t}{2}\,dt$

   $\int \left(1 - \cos\frac{t}{2}\right)^2 \left(\sin\frac{t}{2}\right) dt = \int 2u^2\,du = \frac{2}{3}u^3 + C = \frac{2}{3}\left(1 - \cos\frac{t}{2}\right)^3 + C$

5. Let $u = 7x - 2 \Rightarrow du = 7\,dx \Rightarrow \frac{1}{7}\,du = dx$

   $\int 28(7x-2)^{-5}\,dx = \int \frac{1}{7}(28)u^{-5}\,du = \int 4u^{-5}\,du = -u^{-4} + C = -(7x-2)^{-4} + C$

6. Let $u = x^4 - 1 \Rightarrow du = 4x^3\,dx \Rightarrow \frac{1}{4}\,du = x^3\,dx$

   $\int x^3 (x^4 - 1)^2\,dx = \int \frac{1}{4}u^2\,du = \frac{u^3}{12} + C = \frac{1}{12}(x^4 - 1)^3 + C$

7. Let $u = 1 - r^3 \Rightarrow du = -3r^2\,dr \Rightarrow -3\,du = 9r^2\,dr$
$\int \frac{9r^2\,dr}{\sqrt{1-r^3}} = \int -3u^{-1/2}\,du = -3(2)u^{1/2} + C = -6\left(1-r^3\right)^{1/2} + C$

8. Let $u = y^4 + 4y^2 + 1 \Rightarrow du = (4y^3 + 8y)\,dy \Rightarrow 3\,du = 12\left(y^3 + 2y\right)dy$
$\int 12\left(y^4 + 4y^2 + 1\right)^2\left(y^3 + 2y\right)dy = \int 3u^2\,du = u^3 + C = \left(y^4 + 4y^2 + 1\right)^3 + C$

9. Let $u = x^{3/2} - 1 \Rightarrow du = \frac{3}{2}x^{1/2}\,dx \Rightarrow \frac{2}{3}\,du = \sqrt{x}\,dx$
$\int \sqrt{x}\sin^2\left(x^{3/2} - 1\right)dx = \int \frac{2}{3}\sin^2 u\,du = \frac{2}{3}\left(\frac{u}{2} - \frac{1}{4}\sin 2u\right) + C = \frac{1}{3}\left(x^{3/2} - 1\right) - \frac{1}{6}\sin\left(2x^{3/2} - 2\right) + C$

10. Let $u = -\frac{1}{x} \Rightarrow du = \frac{1}{x^2}\,dx$
$\int \frac{1}{x^2}\cos^2\left(\frac{1}{x}\right)dx = \int \cos^2(-u)\,du = \int \cos^2(u)\,du = \left(\frac{u}{2} + \frac{1}{4}\sin 2u\right) + C = -\frac{1}{2x} + \frac{1}{4}\sin\left(-\frac{2}{x}\right) + C$
$= -\frac{1}{2x} - \frac{1}{4}\sin\left(\frac{2}{x}\right) + C$

11. (a) Let $u = \cot 2\theta \Rightarrow du = -2\csc^2 2\theta\,d\theta \Rightarrow -\frac{1}{2}\,du = \csc^2 2\theta\,d\theta$
$\int \csc^2 2\theta\cot 2\theta\,d\theta = -\int \frac{1}{2}u\,du = -\frac{1}{2}\left(\frac{u^2}{2}\right) + C = -\frac{u^2}{4} + C = -\frac{1}{4}\cot^2 2\theta + C$
(b) Let $u = \csc 2\theta \Rightarrow du = -2\csc 2\theta\cot 2\theta\,d\theta \Rightarrow -\frac{1}{2}\,du = \csc 2\theta\cot 2\theta\,d\theta$
$\int \csc^2 2\theta\cot 2\theta\,d\theta = \int -\frac{1}{2}u\,du = -\frac{1}{2}\left(\frac{u^2}{2}\right) + C = -\frac{u^2}{4} + C = -\frac{1}{4}\csc^2 2\theta + C$

12. (a) Let $u = 5x + 8 \Rightarrow du = 5\,dx \Rightarrow \frac{1}{5}\,du = dx$
$\int \frac{dx}{\sqrt{5x+8}} = \int \frac{1}{5}\left(\frac{1}{\sqrt{u}}\right)du = \frac{1}{5}\int u^{-1/2}\,du = \frac{1}{5}\left(2u^{1/2}\right) + C = \frac{2}{5}u^{1/2} + C = \frac{2}{5}\sqrt{5x+8} + C$
(b) Let $u = \sqrt{5x+8} \Rightarrow du = \frac{1}{2}(5x+8)^{-1/2}(5)\,dx \Rightarrow \frac{2}{5}\,du = \frac{dx}{\sqrt{5x+8}}$
$\int \frac{dx}{\sqrt{5x+8}} = \int \frac{2}{5}\,du = \frac{2}{5}u + C = \frac{2}{5}\sqrt{5x+8} + C$

13. Let $u = 3 - 2s \Rightarrow du = -2\,ds \Rightarrow -\frac{1}{2}\,du = ds$
$\int \sqrt{3-2s}\,ds = \int \sqrt{u}\left(-\frac{1}{2}\,du\right) = -\frac{1}{2}\int u^{1/2}\,du = \left(-\frac{1}{2}\right)\left(\frac{2}{3}u^{3/2}\right) + C = -\frac{1}{3}(3-2s)^{3/2} + C$

14. Let $u = 2x + 1 \Rightarrow du = 2\,dx \Rightarrow \frac{1}{2}\,du = dx$
$\int (2x+1)^3\,dx = \int u^3\left(\frac{1}{2}\,du\right) = \frac{1}{2}\int u^3\,du = \left(\frac{1}{2}\right)\left(\frac{u^4}{4}\right) + C = \frac{1}{8}(2x+1)^4 + C$

15. Let $u = 5s + 4 \Rightarrow du = 5\,ds \Rightarrow \frac{1}{5}\,du = ds$
$\int \frac{1}{\sqrt{5s+4}}\,ds = \int \frac{1}{\sqrt{u}}\left(\frac{1}{5}\,du\right) = \frac{1}{5}\int u^{-1/2}\,du = \left(\frac{1}{5}\right)\left(2u^{1/2}\right) + C = \frac{2}{5}\sqrt{5s+4} + C$

16. Let $u = 2 - x \Rightarrow du = -dx \Rightarrow -du = dx$
$\int \frac{3}{(2-x)^2}\,dx = \int \frac{3(-du)}{u^2} = -3\int u^{-2}\,du = -3\left(\frac{u^{-1}}{-1}\right) + C = \frac{3}{2-x} + C$

17. Let $u = 1 - \theta^2 \Rightarrow du = -2\theta\,d\theta \Rightarrow -\frac{1}{2}\,du = \theta\,d\theta$
$\int \theta\sqrt[4]{1-\theta^2}\,d\theta = \int \sqrt[4]{u}\left(-\frac{1}{2}\,du\right) = -\frac{1}{2}\int u^{1/4}\,du = \left(-\frac{1}{2}\right)\left(\frac{4}{5}u^{5/4}\right) + C = -\frac{2}{5}\left(1-\theta^2\right)^{5/4} + C$

18. Let $u = \theta^2 - 1 \Rightarrow du = 2\theta\,d\theta \Rightarrow 4\,du = 8\theta\,d\theta$
$\int 8\theta\sqrt[3]{\theta^2 - 1}\,d\theta = \int \sqrt[3]{u}\,(4\,du) = 4\int u^{1/3}\,du = 4\left(\frac{3}{4}u^{4/3}\right) + C = 3\left(\theta^2 - 1\right)^{4/3} + C$

19. Let $u = 7 - 3y^2 \Rightarrow du = -6y\,dy \Rightarrow -\frac{1}{2}\,du = 3y\,dy$

$$\int 3y\sqrt{7-3y^2}\,dy = \int \sqrt{u}\left(-\tfrac{1}{2}\,du\right) = -\tfrac{1}{2}\int u^{1/2}\,du = \left(-\tfrac{1}{2}\right)\left(\tfrac{2}{3}u^{3/2}\right) + C = -\tfrac{1}{3}\left(7-3y^2\right)^{3/2} + C$$

20. Let $u = 2y^2 + 1 \Rightarrow du = 4y\,dy$

$$\int \frac{4y\,dy}{\sqrt{2y^2+1}} = \int \frac{1}{\sqrt{u}}\,du = \int u^{-1/2}\,du = 2u^{1/2} + C = 2\sqrt{2y^2+1} + C$$

21. Let $u = 1 + \sqrt{x} \Rightarrow du = \frac{1}{2\sqrt{x}}\,dx \Rightarrow 2\,du = \frac{1}{\sqrt{x}}\,dx$

$$\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}\right)^2}\,dx = \int \frac{2\,du}{u^2} = -\frac{2}{u} + C = \frac{-2}{1+\sqrt{x}} + C$$

22. Let $u = 1 + \sqrt{x} \Rightarrow du = \frac{1}{2\sqrt{x}}\,dx \Rightarrow 2\,du = \frac{1}{\sqrt{x}}\,dx$

$$\int \frac{\left(1+\sqrt{x}\right)^3}{\sqrt{x}}\,dx = \int u^3\,(2\,du) = 2\left(\tfrac{1}{4}u^4\right) + C = \tfrac{1}{2}\left(1+\sqrt{x}\right)^4 + C$$

23. Let $u = 3z + 4 \Rightarrow du = 3\,dz \Rightarrow \frac{1}{3}\,du = dz$

$$\int \cos(3z+4)\,dz = \int (\cos u)\left(\tfrac{1}{3}\,du\right) = \tfrac{1}{3}\int \cos u\,du = \tfrac{1}{3}\sin u + C = \tfrac{1}{3}\sin(3z+4) + C$$

24. Let $u = 8z - 5 \Rightarrow du = 8\,dz \Rightarrow \frac{1}{8}\,du = dz$

$$\int \sin(8z-5)\,dz = \int (\sin u)\left(\tfrac{1}{8}\,du\right) = \tfrac{1}{8}\int \sin u\,du = \tfrac{1}{8}(-\cos u) + C = -\tfrac{1}{8}\cos(8z-5) + C$$

25. Let $u = 3x + 2 \Rightarrow du = 3\,dx \Rightarrow \frac{1}{3}\,du = dx$

$$\int \sec^2(3x+2)\,dx = \int (\sec^2 u)\left(\tfrac{1}{3}\,du\right) = \tfrac{1}{3}\int \sec^2 u\,du = \tfrac{1}{3}\tan u + C = \tfrac{1}{3}\tan(3x+2) + C$$

26. Let $u = \tan x \Rightarrow du = \sec^2 x\,dx$

$$\int \tan^2 x \sec^2 x\,dx = \int u^2\,du = \tfrac{1}{3}u^3 + C = \tfrac{1}{3}\tan^3 x + C$$

27. Let $u = \sin\left(\frac{x}{3}\right) \Rightarrow du = \frac{1}{3}\cos\left(\frac{x}{3}\right)dx \Rightarrow 3\,du = \cos\left(\frac{x}{3}\right)dx$

$$\int \sin^5\left(\tfrac{x}{3}\right)\cos\left(\tfrac{x}{3}\right)dx = \int u^5\,(3\,du) = 3\left(\tfrac{1}{6}u^6\right) + C = \tfrac{1}{2}\sin^6\left(\tfrac{x}{3}\right) + C$$

28. Let $u = \tan\left(\frac{x}{2}\right) \Rightarrow du = \frac{1}{2}\sec^2\left(\frac{x}{2}\right)dx \Rightarrow 2\,du = \sec^2\left(\frac{x}{2}\right)dx$

$$\int \tan^7\left(\tfrac{x}{2}\right)\sec^2\left(\tfrac{x}{2}\right)dx = \int u^7\,(2\,du) = 2\left(\tfrac{1}{8}u^8\right) + C = \tfrac{1}{4}\tan^8\left(\tfrac{x}{2}\right) + C$$

29. Let $u = \frac{r^3}{18} - 1 \Rightarrow du = \frac{r^2}{6}\,dr \Rightarrow 6\,du = r^2\,dr$

$$\int r^2\left(\frac{r^3}{18} - 1\right)^5 dr = \int u^5\,(6\,du) = 6\int u^5\,du = 6\left(\frac{u^6}{6}\right) + C = \left(\frac{r^3}{18} - 1\right)^6 + C$$

30. Let $u = 7 - \frac{r^5}{10} \Rightarrow du = -\frac{1}{2}r^4\,dr \Rightarrow -2\,du = r^4\,dr$

$$\int r^4\left(7 - \frac{r^5}{10}\right)^3 dr = \int u^3\,(-2\,du) = -2\int u^3\,du = -2\left(\frac{u^4}{4}\right) + C = -\frac{1}{2}\left(7 - \frac{r^5}{10}\right)^4 + C$$

31. Let $u = x^{3/2} + 1 \Rightarrow du = \frac{3}{2}x^{1/2}\,dx \Rightarrow \frac{2}{3}\,du = x^{1/2}\,dx$

$$\int x^{1/2}\sin\left(x^{3/2}+1\right)dx = \int (\sin u)\left(\tfrac{2}{3}\,du\right) = \tfrac{2}{3}\int \sin u\,du = \tfrac{2}{3}(-\cos u) + C = -\tfrac{2}{3}\cos\left(x^{3/2}+1\right) + C$$

32. Let $u = x^{4/3} - 8 \Rightarrow du = \frac{4}{3} x^{1/3}\, dx \Rightarrow \frac{3}{4}\, du = x^{1/3}\, dx$

$\int x^{1/3} \sin\left(x^{4/3} - 8\right) dx = \int (\sin u)\left(\frac{3}{4}\, du\right) = \frac{3}{4} \int \sin u\, du = \frac{3}{4}(-\cos u) + C = -\frac{3}{4}\cos\left(x^{4/3} - 8\right) + C$

33. Let $u = \sec\left(v + \frac{\pi}{2}\right) \Rightarrow du = \sec\left(v + \frac{\pi}{2}\right)\tan\left(v + \frac{\pi}{2}\right) dv$

$\int \sec\left(v + \frac{\pi}{2}\right)\tan\left(v + \frac{\pi}{2}\right) dv = \int du = u + C = \sec\left(v + \frac{\pi}{2}\right) + C$

34. Let $u = \csc\left(\frac{v-\pi}{2}\right) \Rightarrow du = -\frac{1}{2}\csc\left(\frac{v-\pi}{2}\right)\cot\left(\frac{v-\pi}{2}\right) dv \Rightarrow -2\, du = \csc\left(\frac{v-\pi}{2}\right)\cot\left(\frac{v-\pi}{2}\right) dv$

$\int \csc\left(\frac{v-\pi}{2}\right)\cot\left(\frac{v-\pi}{2}\right) dv = \int -2\, du = -2u + C = -2\csc\left(\frac{v-\pi}{2}\right) + C$

35. Let $u = \cos(2t+1) \Rightarrow du = -2\sin(2t+1)\, dt \Rightarrow -\frac{1}{2}\, du = \sin(2t+1)\, dt$

$\int \frac{\sin(2t+1)}{\cos^2(2t+1)}\, dt = \int -\frac{1}{2}\frac{du}{u^2} = \frac{1}{2u} + C = \frac{1}{2\cos(2t+1)} + C$

36. Let $u = 2 + \sin t \Rightarrow du = \cos t\, dt$

$\int \frac{6\cos t}{(2+\sin t)^3}\, dt = \int \frac{6}{u^3}\, du = 6\int u^{-3}\, du = 6\left(\frac{u^{-2}}{-2}\right) + C = -3(2+\sin t)^{-2} + C$

37. Let $u = \cot y \Rightarrow du = -\csc^2 y\, dy \Rightarrow -du = \csc^2 y\, dy$

$\int \sqrt{\cot y}\,\csc^2 y\, dy = \int \sqrt{u}\,(-du) = -\int u^{1/2}\, du = -\frac{2}{3}u^{3/2} + C = -\frac{2}{3}(\cot y)^{3/2} + C = -\frac{2}{3}\left(\cot^3 y\right)^{1/2} + C$

38. Let $u = \sec z \Rightarrow du = \sec z \tan z\, dz$

$\int \frac{\sec z \tan z}{\sqrt{\sec z}}\, dz = \int \frac{1}{\sqrt{u}}\, du = \int u^{-1/2}\, du = 2u^{1/2} + C = 2\sqrt{\sec z} + C$

39. Let $u = \frac{1}{t} - 1 = t^{-1} - 1 \Rightarrow du = -t^{-2}\, dt \Rightarrow -du = \frac{1}{t^2}\, dt$

$\int \frac{1}{t^2}\cos\left(\frac{1}{t} - 1\right) dt = \int (\cos u)(-du) = -\int \cos u\, du = -\sin u + C = -\sin\left(\frac{1}{t} - 1\right) + C$

40. Let $u = \sqrt{t} + 3 = t^{1/2} + 3 \Rightarrow du = \frac{1}{2}t^{-1/2}\, dt \Rightarrow 2\, du = \frac{1}{\sqrt{t}}\, dt$

$\int \frac{1}{\sqrt{t}}\cos\left(\sqrt{t} + 3\right) dt = \int (\cos u)(2\, du) = 2\int \cos u\, du = 2\sin u + C = 2\sin\left(\sqrt{t} + 3\right) + C$

41. Let $u = \sin\frac{1}{\theta} \Rightarrow du = \left(\cos\frac{1}{\theta}\right)\left(-\frac{1}{\theta^2}\right) d\theta \Rightarrow -du = \frac{1}{\theta^2}\cos\frac{1}{\theta}\, d\theta$

$\int \frac{1}{\theta^2}\sin\frac{1}{\theta}\cos\frac{1}{\theta}\, d\theta = \int -u\, du = -\frac{1}{2}u^2 + C = -\frac{1}{2}\sin^2\frac{1}{\theta} + C$

42. Let $u = \csc\sqrt{\theta} \Rightarrow du = \left(-\csc\sqrt{\theta}\cot\sqrt{\theta}\right)\left(\frac{1}{2\sqrt{\theta}}\right) d\theta \Rightarrow -2\, du = \frac{1}{\sqrt{\theta}}\cot\sqrt{\theta}\csc\sqrt{\theta}\, d\theta$

$\int \frac{\cos\sqrt{\theta}}{\sqrt{\theta}\sin^2\sqrt{\theta}}\, d\theta = \int \frac{1}{\sqrt{\theta}}\cot\sqrt{\theta}\csc\sqrt{\theta}\, d\theta = \int -2\, du = -2u + C = -2\csc\sqrt{\theta} + C = -\frac{2}{\sin\sqrt{\theta}} + C$

43. Let $u = s^3 + 2s^2 - 5s + 5 \Rightarrow du = (3s^2 + 4s - 5)\, ds$

$\int (s^3 + 2s^2 - 5s + 5)(3s^2 + 4s - 5)\, ds = \int u\, du = \frac{u^2}{2} + C = \frac{(s^3 + 2s^2 - 5s + 5)^2}{2} + C$

44. Let $u = \theta^4 - 2\theta^2 + 8\theta - 2 \Rightarrow du = (4\theta^3 - 4\theta + 8)\, d\theta \Rightarrow \frac{1}{4}\, du = (\theta^3 - \theta + 2)\, d\theta$

$\int (\theta^4 - 2\theta^2 + 8\theta - 2)(\theta^3 - \theta + 2)\, d\theta = \int u\left(\frac{1}{4}\, du\right) = \frac{1}{4}\int u\, du = \frac{1}{4}\left(\frac{u^2}{2}\right) + C = \frac{(\theta^4 - 2\theta^2 + 8\theta - 2)^2}{8} + C$

45. Let $u = 1 + t^4 \Rightarrow du = 4t^3\,dt \Rightarrow \frac{1}{4}\,du = t^3\,dt$

$\int t^3\left(1+t^4\right)^3 dt = \int u^3\left(\frac{1}{4}\,du\right) = \frac{1}{4}\left(\frac{1}{4}u^4\right) + C = \frac{1}{16}\left(1+t^4\right)^4 + C$

46. Let $u = 1 - \frac{1}{x} \Rightarrow du = \frac{1}{x^2}\,dx$

$\int \sqrt{\frac{x-1}{x^5}}\,dx = \int \frac{1}{x^2}\sqrt{\frac{x-1}{x}}\,dx = \int \frac{1}{x^2}\sqrt{1-\frac{1}{x}}\,dx = \int \sqrt{u}\,du = \int u^{1/2}\,du = \frac{2}{3}u^{3/2} + C = \frac{2}{3}\left(1-\frac{1}{x}\right)^{3/2} + C$

47. Let $u = x^2 + 1$. Then $du = 2x\,dx$ and $\frac{1}{2}du = x\,dx$ and $x^2 = u - 1$. Thus $\int x^3\sqrt{x^2+1}\,dx = \int (u-1)\frac{1}{2}\sqrt{u}\,du$

$= \frac{1}{2}\int\left(u^{3/2} - u^{1/2}\right)du = \frac{1}{2}\left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right] + C = \frac{1}{5}u^{5/2} - \frac{1}{3}u^{3/2} + C = \frac{1}{5}(x^2+1)^{5/2} - \frac{1}{3}(x^2+1)^{3/2} + C$

48. Let $u = x^3 + 1 \Rightarrow du = 3x^2dx$ and $x^3 = u - 1$. So $\int 3x^5\sqrt{x^3+1}\,dx = \int (u-1)\sqrt{u}\,du = \int\left(u^{3/2} - u^{1/2}\right)du$

$= \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + C = \frac{2}{5}(x^3+1)^{5/2} - \frac{2}{3}(x^3+1)^{3/2} + C$

49. (a) Let $u = \tan x \Rightarrow du = \sec^2 x\,dx$; $v = u^3 \Rightarrow dv = 3u^2\,du \Rightarrow 6\,dv = 18u^2\,du$; $w = 2 + v \Rightarrow dw = dv$

$\int \frac{18\tan^2 x\sec^2 x}{\left(2+\tan^3 x\right)^2}\,dx = \int \frac{18u^2}{\left(2+u^3\right)^2}\,du = \int \frac{6\,dv}{(2+v)^2} = \int \frac{6\,dw}{w^2} = 6\int w^{-2}\,dw = -6w^{-1} + C = -\frac{6}{2+v} + C$

$= -\frac{6}{2+u^3} + C = -\frac{6}{2+\tan^3 x} + C$

(b) Let $u = \tan^3 x \Rightarrow du = 3\tan^2 x\sec^2 x\,dx \Rightarrow 6\,du = 18\tan^2 x\sec^2 x\,dx$; $v = 2 + u \Rightarrow dv = du$

$\int \frac{18\tan^2 x\sec^2 x}{\left(2+\tan^3 x\right)^2}\,dx = \int \frac{6\,du}{(2+u)^2} = \int \frac{6\,dv}{v^2} = -\frac{6}{v} + C = -\frac{6}{2+u} + C = -\frac{6}{2+\tan^3 x} + C$

(c) Let $u = 2 + \tan^3 x \Rightarrow du = 3\tan^2 x\sec^2 x\,dx \Rightarrow 6\,du = 18\tan^2 x\sec^2 x\,dx$

$\int \frac{18\tan^2 x\sec^2 x}{\left(2+\tan^3 x\right)^2}\,dx = \int \frac{6\,du}{u^2} = -\frac{6}{u} + C = -\frac{6}{2+\tan^3 x} + C$

50. (a) Let $u = x - 1 \Rightarrow du = dx$; $v = \sin u \Rightarrow dv = \cos u\,du$; $w = 1 + v^2 \Rightarrow dw = 2v\,dv \Rightarrow \frac{1}{2}\,dw = v\,dv$

$\int \sqrt{1+\sin^2(x-1)}\,\sin(x-1)\cos(x-1)\,dx = \int \sqrt{1+\sin^2 u}\,\sin u\cos u\,du = \int v\sqrt{1+v^2}\,dv$

$= \int \frac{1}{2}\sqrt{w}\,dw = \frac{1}{3}w^{3/2} + C = \frac{1}{3}\left(1+v^2\right)^{3/2} + C = \frac{1}{3}\left(1+\sin^2 u\right)^{3/2} + C = \frac{1}{3}\left(1+\sin^2(x-1)\right)^{3/2} + C$

(b) Let $u = \sin(x-1) \Rightarrow du = \cos(x-1)\,dx$; $v = 1 + u^2 \Rightarrow dv = 2u\,du \Rightarrow \frac{1}{2}\,dv = u\,du$

$\int \sqrt{1+\sin^2(x-1)}\,\sin(x-1)\cos(x-1)\,dx = \int u\sqrt{1+u^2}\,du = \int \frac{1}{2}\sqrt{v}\,dv = \int \frac{1}{2}v^{1/2}\,dv$

$= \left(\frac{1}{2}\left(\frac{2}{3}\right)v^{3/2}\right) + C = \frac{1}{3}v^{3/2} + C = \frac{1}{3}\left(1+u^2\right)^{3/2} + C = \frac{1}{3}\left(1+\sin^2(x-1)\right)^{3/2} + C$

(c) Let $u = 1 + \sin^2(x-1) \Rightarrow du = 2\sin(x-1)\cos(x-1)\,dx \Rightarrow \frac{1}{2}\,du = \sin(x-1)\cos(x-1)\,dx$

$\int \sqrt{1+\sin^2(x-1)}\,\sin(x-1)\cos(x-1)\,dx = \int \frac{1}{2}\sqrt{u}\,du = \int \frac{1}{2}u^{1/2}\,du = \frac{1}{2}\left(\frac{2}{3}u^{3/2}\right) + C$

$= \frac{1}{3}\left(1+\sin^2(x-1)\right)^{3/2} + C$

51. Let $u = 3(2r-1)^2 + 6 \Rightarrow du = 6(2r-1)(2)\,dr \Rightarrow \frac{1}{12}\,du = (2r-1)\,dr$; $v = \sqrt{u} \Rightarrow dv = \frac{1}{2\sqrt{u}}\,du \Rightarrow \frac{1}{6}\,dv$

$= \frac{1}{12\sqrt{u}}\,du$

$\int \frac{(2r-1)\cos\sqrt{3(2r-1)^2+6}}{\sqrt{3(2r-1)^2+6}}\,dr = \int\left(\frac{\cos\sqrt{u}}{\sqrt{u}}\right)\left(\frac{1}{12}\,du\right) = \int(\cos v)\left(\frac{1}{6}\,dv\right) = \frac{1}{6}\sin v + C = \frac{1}{6}\sin\sqrt{u} + C$

$= \frac{1}{6}\sin\sqrt{3(2r-1)^2+6} + C$

52. Let $u = \cos\sqrt{\theta} \Rightarrow du = \left(-\sin\sqrt{\theta}\right)\left(\frac{1}{2\sqrt{\theta}}\right)d\theta \Rightarrow -2\,du = \frac{\sin\sqrt{\theta}}{\sqrt{\theta}}\,d\theta$

$\int \frac{\sin\sqrt{\theta}}{\sqrt{\theta\cos^3\sqrt{\theta}}}\,d\theta = \int \frac{\sin\sqrt{\theta}}{\sqrt{\theta}\sqrt{\cos^3\sqrt{\theta}}}\,d\theta = \int \frac{-2\,du}{u^{3/2}} = -2\int u^{-3/2}\,du = -2\left(-2u^{-1/2}\right) + C = \frac{4}{\sqrt{u}} + C$

$= \frac{4}{\sqrt{\cos\sqrt{\theta}}} + C$

53. Let $u = 3t^2 - 1 \Rightarrow du = 6t\,dt \Rightarrow 2\,du = 12t\,dt$
$s = \int 12t\,(3t^2-1)^3\,dt = \int u^3\,(2\,du) = 2\left(\frac{1}{4}u^4\right) + C = \frac{1}{2}u^4 + C = \frac{1}{2}(3t^2-1)^4 + C;$
$s = 3$ when $t = 1 \Rightarrow 3 = \frac{1}{2}(3-1)^4 + C \Rightarrow 3 = 8 + C \Rightarrow C = -5 \Rightarrow s = \frac{1}{2}(3t^2-1)^4 - 5$

54. Let $u = x^2 + 8 \Rightarrow du = 2x\,dx \Rightarrow 2\,du = 4x\,dx$
$y = \int 4x\,(x^2+8)^{-1/3}\,dx = \int u^{-1/3}\,(2\,du) = 2\left(\frac{3}{2}u^{2/3}\right) + C = 3u^{2/3} + C = 3\,(x^2+8)^{2/3} + C;$
$y = 0$ when $x = 0 \Rightarrow 0 = 3(8)^{2/3} + C \Rightarrow C = -12 \Rightarrow y = 3\,(x^2+8)^{2/3} - 12$

55. Let $u = t + \frac{\pi}{12} \Rightarrow du = dt$
$s = \int 8\sin^2\left(t + \frac{\pi}{12}\right)dt = \int 8\sin^2 u\,du = 8\left(\frac{u}{2} - \frac{1}{4}\sin 2u\right) + C = 4\left(t + \frac{\pi}{12}\right) - 2\sin\left(2t + \frac{\pi}{6}\right) + C;$
$s = 8$ when $t = 0 \Rightarrow 8 = 4\left(\frac{\pi}{12}\right) - 2\sin\left(\frac{\pi}{6}\right) + C \Rightarrow C = 8 - \frac{\pi}{3} + 1 = 9 - \frac{\pi}{3}$
$\Rightarrow s = 4\left(t + \frac{\pi}{12}\right) - 2\sin\left(2t + \frac{\pi}{6}\right) + 9 - \frac{\pi}{3} = 4t - 2\sin\left(2t + \frac{\pi}{6}\right) + 9$

56. Let $u = \frac{\pi}{4} - \theta \Rightarrow -du = d\theta$
$r = \int 3\cos^2\left(\frac{\pi}{4} - \theta\right)d\theta = -\int 3\cos^2 u\,du = -3\left(\frac{u}{2} + \frac{1}{4}\sin 2u\right) + C = -\frac{3}{2}\left(\frac{\pi}{4} - \theta\right) - \frac{3}{4}\sin\left(\frac{\pi}{2} - 2\theta\right) + C;$
$r = \frac{\pi}{8}$ when $\theta = 0 \Rightarrow \frac{\pi}{8} = -\frac{3\pi}{8} - \frac{3}{4}\sin\frac{\pi}{2} + C \Rightarrow C = \frac{\pi}{2} + \frac{3}{4} \Rightarrow r = -\frac{3}{2}\left(\frac{\pi}{4} - \theta\right) - \frac{3}{4}\sin\left(\frac{\pi}{2} - 2\theta\right) + \frac{\pi}{2} + \frac{3}{4}$
$\Rightarrow r = \frac{3}{2}\theta - \frac{3}{4}\sin\left(\frac{\pi}{2} - 2\theta\right) + \frac{\pi}{8} + \frac{3}{4} \Rightarrow r = \frac{3}{2}\theta - \frac{3}{4}\cos 2\theta + \frac{\pi}{8} + \frac{3}{4}$

57. Let $u = 2t - \frac{\pi}{2} \Rightarrow du = 2\,dt \Rightarrow -2\,du = -4\,dt$
$\frac{ds}{dt} = \int -4\sin\left(2t - \frac{\pi}{2}\right)dt = \int (\sin u)(-2\,du) = 2\cos u + C_1 = 2\cos\left(2t - \frac{\pi}{2}\right) + C_1;$
at $t = 0$ and $\frac{ds}{dt} = 100$ we have $100 = 2\cos\left(-\frac{\pi}{2}\right) + C_1 \Rightarrow C_1 = 100 \Rightarrow \frac{ds}{dt} = 2\cos\left(2t - \frac{\pi}{2}\right) + 100$
$\Rightarrow s = \int \left(2\cos\left(2t - \frac{\pi}{2}\right) + 100\right)dt = \int (\cos u + 50)\,du = \sin u + 50u + C_2 = \sin\left(2t - \frac{\pi}{2}\right) + 50\left(2t - \frac{\pi}{2}\right) + C_2;$
at $t = 0$ and $s = 0$ we have $0 = \sin\left(-\frac{\pi}{2}\right) + 50\left(-\frac{\pi}{2}\right) + C_2 \Rightarrow C_2 = 1 + 25\pi$
$\Rightarrow s = \sin\left(2t - \frac{\pi}{2}\right) + 100t - 25\pi + (1 + 25\pi) \Rightarrow s = \sin\left(2t - \frac{\pi}{2}\right) + 100t + 1$

58. Let $u = \tan 2x \Rightarrow du = 2\sec^2 2x\,dx \Rightarrow 2\,du = 4\sec^2 2x\,dx;\ v = 2x \Rightarrow dv = 2\,dx \Rightarrow \frac{1}{2}\,dv = dx$
$\frac{dy}{dx} = \int 4\sec^2 2x\tan 2x\,dx = \int u(2\,du) = u^2 + C_1 = \tan^2 2x + C_1;$
at $x = 0$ and $\frac{dy}{dx} = 4$ we have $4 = 0 + C_1 \Rightarrow C_1 = 4 \Rightarrow \frac{dy}{dx} = \tan^2 2x + 4 = (\sec^2 2x - 1) + 4 = \sec^2 2x + 3$
$\Rightarrow y = \int (\sec^2 2x + 3)\,dx = \int (\sec^2 v + 3)\left(\frac{1}{2}\,dv\right) = \frac{1}{2}\tan v + \frac{3}{2}v + C_2 = \frac{1}{2}\tan 2x + 3x + C_2;$
at $x = 0$ and $y = -1$ we have $-1 = \frac{1}{2}(0) + 0 + C_2 \Rightarrow C_2 = -1 \Rightarrow y = \frac{1}{2}\tan 2x + 3x - 1$

59. Let $u = 2t \Rightarrow du = 2\,dt \Rightarrow 3\,du = 6\,dt$
$s = \int 6\sin 2t\,dt = \int (\sin u)(3\,du) = -3\cos u + C = -3\cos 2t + C;$
at $t = 0$ and $s = 0$ we have $0 = -3\cos 0 + C \Rightarrow C = 3 \Rightarrow s = 3 - 3\cos 2t \Rightarrow s\left(\frac{\pi}{2}\right) = 3 - 3\cos(\pi) = 6$ m

60. Let $u = \pi t \Rightarrow du = \pi\,dt \Rightarrow \pi\,du = \pi^2\,dt$
$v = \int \pi^2\cos\pi t\,dt = \int (\cos u)(\pi\,du) = \pi\sin u + C_1 = \pi\sin(\pi t) + C_1;$
at $t = 0$ and $v = 8$ we have $8 = \pi(0) + C_1 \Rightarrow C_1 = 8 \Rightarrow v = \frac{ds}{dt} = \pi\sin(\pi t) + 8 \Rightarrow s = \int (\pi\sin(\pi t) + 8)\,dt$
$= \int \sin u\,du + 8t + C_2 = -\cos(\pi t) + 8t + C_2;$ at $t = 0$ and $s = 0$ we have $0 = -1 + C_2 \Rightarrow C_2 = 1$