

EXERCISES 5.5

Evaluating Integrals

Evaluate the indefinite integrals in Exercises 1–12 by using the given substitutions to reduce the integrals to standard form.

1. $\int \sin 3x \, dx, \quad u = 3x$

2. $\int x \sin (2x^2) \, dx, \quad u = 2x^2$

3. $\int \sec 2t \tan 2t \, dt, \quad u = 2t$

4. $\int \left(1 - \cos \frac{t}{2}\right)^2 \sin \frac{t}{2} \, dt, \quad u = 1 - \cos \frac{t}{2}$

5. $\int 28(7x - 2)^{-5} dx, \quad u = 7x - 2$

6. $\int x^3(x^4 - 1)^2 dx, \quad u = x^4 - 1$

7. $\int \frac{9r^2 dr}{\sqrt{1 - r^3}}, \quad u = 1 - r^3$

8. $\int 12(y^4 + 4y^2 + 1)^2(y^3 + 2y) dy, \quad u = y^4 + 4y^2 + 1$

9. $\int \sqrt{x} \sin^2(x^{3/2} - 1) dx, \quad u = x^{3/2} - 1$

10. $\int \frac{1}{x^2} \cos^2\left(\frac{1}{x}\right) dx, \quad u = -\frac{1}{x}$

11. $\int \csc^2 2\theta \cot 2\theta d\theta$

a. Using $u = \cot 2\theta$ b. Using $u = \csc 2\theta$

12. $\int \frac{dx}{\sqrt{5x + 8}}$

a. Using $u = 5x + 8$ b. Using $u = \sqrt{5x + 8}$

Evaluate the integrals in Exercises 13–48.

13. $\int \sqrt{3 - 2s} ds$

14. $\int (2x + 1)^3 dx$

15. $\int \frac{1}{\sqrt{5s + 4}} ds$

16. $\int \frac{3 dx}{(2 - x)^2}$

17. $\int \theta \sqrt[4]{1 - \theta^2} d\theta$

18. $\int 8\theta \sqrt[3]{\theta^2 - 1} d\theta$

19. $\int 3y \sqrt{7 - 3y^2} dy$

20. $\int \frac{4y dy}{\sqrt{2y^2 + 1}}$

21. $\int \frac{1}{\sqrt{x}(1 + \sqrt{x})^2} dx$

22. $\int \frac{(1 + \sqrt{x})^3}{\sqrt{x}} dx$

23. $\int \cos(3z + 4) dz$

24. $\int \sin(8z - 5) dz$

25. $\int \sec^2(3x + 2) dx$

26. $\int \tan^2 x \sec^2 x dx$

27. $\int \sin^5 \frac{x}{3} \cos \frac{x}{3} dx$

28. $\int \tan^7 \frac{x}{2} \sec^2 \frac{x}{2} dx$

29. $\int r^2 \left(\frac{r^3}{18} - 1\right)^5 dr$

30. $\int r^4 \left(7 - \frac{r^5}{10}\right)^3 dr$

31. $\int x^{1/2} \sin(x^{3/2} + 1) dx$

32. $\int x^{1/3} \sin(x^{4/3} - 8) dx$

33. $\int \sec\left(v + \frac{\pi}{2}\right) \tan\left(v + \frac{\pi}{2}\right) dv$

34. $\int \csc\left(\frac{v - \pi}{2}\right) \cot\left(\frac{v - \pi}{2}\right) dv$

35. $\int \frac{\sin(2t + 1)}{\cos^2(2t + 1)} dt$

36. $\int \frac{6 \cos t}{(2 + \sin t)^3} dt$

37. $\int \sqrt{\cot y} \csc^2 y dy$

38. $\int \frac{\sec z \tan z}{\sqrt{\sec z}} dz$

39. $\int \frac{1}{t^2} \cos\left(\frac{1}{t} - 1\right) dt$

40. $\int \frac{1}{\sqrt{t}} \cos(\sqrt{t} + 3) dt$

41. $\int \frac{1}{\theta^2} \sin \frac{1}{\theta} \cos \frac{1}{\theta} d\theta$

42. $\int \frac{\cos \sqrt{\theta}}{\sqrt{\theta} \sin^2 \sqrt{\theta}} d\theta$

43. $\int (s^3 + 2s^2 - 5s + 5)(3s^2 + 4s - 5) ds$

44. $\int (\theta^4 - 2\theta^2 + 8\theta - 2)(\theta^3 - \theta + 2) d\theta$

45. $\int t^3(1 + t^4)^3 dt$

46. $\int \sqrt{\frac{x-1}{x^5}} dx$

47. $\int x^3 \sqrt{x^2 + 1} dx$

48. $\int 3x^5 \sqrt{x^3 + 1} dx$

Simplifying Integrals Step by Step

If you do not know what substitution to make, try reducing the integral step by step, using a trial substitution to simplify the integral a bit and then another to simplify it some more. You will see what we mean if you try the sequences of substitutions in Exercises 49 and 50.

49. $\int \frac{18 \tan^2 x \sec^2 x}{(2 + \tan^3 x)^2} dx$

a. $u = \tan x$, followed by $v = u^3$, then by $w = 2 + v$ b. $u = \tan^3 x$, followed by $v = 2 + u$ c. $u = 2 + \tan^3 x$

50. $\int \sqrt{1 + \sin^2(x - 1)} \sin(x - 1) \cos(x - 1) dx$

a. $u = x - 1$, followed by $v = \sin u$, then by $w = 1 + v^2$ b. $u = \sin(x - 1)$, followed by $v = 1 + u^2$ c. $u = 1 + \sin^2(x - 1)$

Evaluate the integrals in Exercises 51 and 52.

51. $\int \frac{(2r - 1) \cos \sqrt{3(2r - 1)^2 + 6}}{\sqrt{3(2r - 1)^2 + 6}} dr$

52. $\int \frac{\sin \sqrt{\theta}}{\sqrt{\theta} \cos^3 \sqrt{\theta}} d\theta$

Initial Value Problems

Solve the initial value problems in Exercises 53–58.

53. $\frac{ds}{dt} = 12t(3t^2 - 1)^3, \quad s(1) = 3$

54. $\frac{dy}{dx} = 4x(x^2 + 8)^{-1/3}, \quad y(0) = 0$

55. $\frac{ds}{dt} = 8 \sin^2\left(t + \frac{\pi}{12}\right), \quad s(0) = 8$

56. $\frac{dr}{d\theta} = 3 \cos^2\left(\frac{\pi}{4} - \theta\right), \quad r(0) = \frac{\pi}{8}$