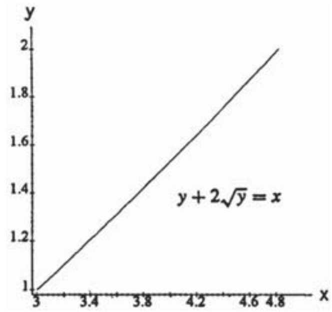
6. (a) $\frac{dx}{dy} = 1 + y^{-1/2} \Rightarrow \left(\frac{dx}{dy}\right)^2 = \left(1 + y^{-1/2}\right)^2$

$\Rightarrow S = 2\pi \int_1^2 \left(y + 2\sqrt{y}\right) \sqrt{1 + \left(1 + y^{-1/2}\right)^2}\, dx$

(b)



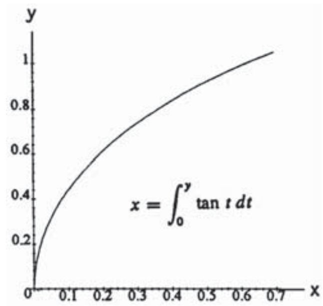
(c) $S \approx 51.33$

7. (a) $\frac{dx}{dy} = \tan y \Rightarrow \left(\frac{dx}{dy}\right)^2 = \tan^2 y$

$\Rightarrow S = 2\pi \int_0^{\pi/3} \left(\int_0^y \tan t\, dt\right) \sqrt{1 + \tan^2 y}\, dy$

$= 2\pi \int_0^{\pi/3} \left(\int_0^y \tan t\, dt\right) \sec y\, dy$

(b)



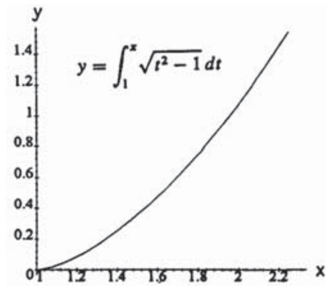
(c) $S \approx 2.08$

8. (a) $\frac{dy}{dx} = \sqrt{x^2 - 1} \Rightarrow \left(\frac{dy}{dx}\right)^2 = x^2 - 1$

$\Rightarrow S = 2\pi \int_1^{\sqrt{5}} \left(\int_1^x \sqrt{t^2 - 1}\, dt\right) \sqrt{1 + (x^2 - 1)}\, dx$

$= 2\pi \int_1^{\sqrt{5}} \left(\int_1^x \sqrt{t^2 - 1}\, dt\right) x\, dx$

(b)



(c) $S \approx 8.55$

9. $y = \frac{x}{2} \Rightarrow \frac{dy}{dx} = \frac{1}{2}$; $S = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \Rightarrow S = \int_0^4 2\pi \left(\frac{x}{2}\right) \sqrt{1 + \frac{1}{4}}\, dx = \frac{\pi\sqrt{5}}{2} \int_0^4 x\, dx$

$= \frac{\pi\sqrt{5}}{2} \left[\frac{x^2}{2}\right]_0^4 = 4\pi\sqrt{5}$; Geometry formula: base circumference $= 2\pi(2)$, slant height $= \sqrt{4^2 + 2^2} = 2\sqrt{5}$

$\Rightarrow$ Lateral surface area $= \frac{1}{2}(4\pi)\left(2\sqrt{5}\right) = 4\pi\sqrt{5}$ in agreement with the integral value

10. $y = \frac{x}{2} \Rightarrow x = 2y \Rightarrow \frac{dx}{dy} = 2$; $S = \int_c^d 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy = \int_0^2 2\pi \cdot 2y \sqrt{1 + 2^2}\, dy = 4\pi\sqrt{5} \int_0^2 y\, dy = 2\pi\sqrt{5}\left[y^2\right]_0^2$

$= 2\pi\sqrt{5} \cdot 4 = 8\pi\sqrt{5}$; Geometry formula: base circumference $= 2\pi(4)$, slant height $= \sqrt{4^2 + 2^2} = 2\sqrt{5}$

$\Rightarrow$ Lateral surface area $= \frac{1}{2}(8\pi)\left(2\sqrt{5}\right) = 8\pi\sqrt{5}$ in agreement with the integral value

11. $\frac{dy}{dx} = \frac{1}{2}$; $S = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \int_1^3 2\pi \frac{(x+1)}{2} \sqrt{1 + \left(\frac{1}{2}\right)^2}\, dx = \frac{\pi\sqrt{5}}{2} \int_1^3 (x+1)\, dx = \frac{\pi\sqrt{5}}{2} \left[\frac{x^2}{2} + x\right]_1^3$

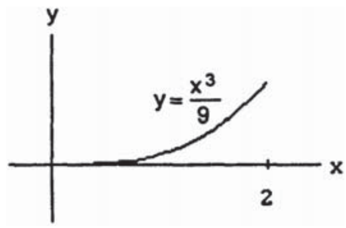
$= \frac{\pi\sqrt{5}}{2} \left[\left(\frac{9}{2} + 3\right) - \left(\frac{1}{2} + 1\right)\right] = \frac{\pi\sqrt{5}}{2}(4 + 2) = 3\pi\sqrt{5}$; Geometry formula: $r_1 = \frac{1}{2} + \frac{1}{2} = 1$, $r_2 = \frac{3}{2} + \frac{1}{2} = 2$,

slant height $= \sqrt{(2-1)^2 + (3-1)^2} = \sqrt{5} \Rightarrow$ Frustum surface area $= \pi(r_1 + r_2) \times$ slant height $= \pi(1 + 2)\sqrt{5}$

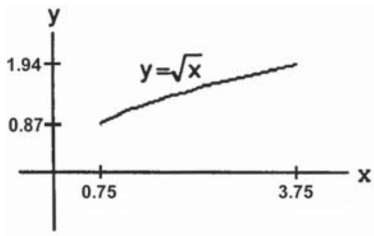
$= 3\pi\sqrt{5}$ in agreement with the integral value

12. $y = \frac{x}{2} + \frac{1}{2} \Rightarrow x = 2y - 1 \Rightarrow \frac{dx}{dy} = 2$; $S = \int_c^d 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy = \int_1^2 2\pi(2y - 1)\sqrt{1 + 4}\, dy = 2\pi\sqrt{5}\int_1^2 (2y - 1)\, dy$
$= 2\pi\sqrt{5}\,[y^2 - y]_1^2 = 2\pi\sqrt{5}\,[(4 - 2) - (1 - 1)] = 4\pi\sqrt{5}$; Geometry formula: $r_1 = 1$, $r_2 = 3$, slant height $= \sqrt{(2 - 1)^2 + (3 - 1)^2} = \sqrt{5} \Rightarrow$ Frustum surface area $= \pi(1 + 3)\sqrt{5} = 4\pi\sqrt{5}$ in agreement with the integral value

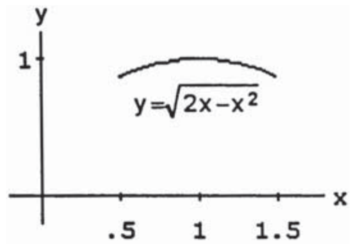
13. $\frac{dy}{dx} = \frac{x^2}{3} \Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{x^4}{9} \Rightarrow S = \int_0^2 \frac{2\pi x^3}{9}\sqrt{1 + \frac{x^4}{9}}\, dx$;
$\left[u = 1 + \frac{x^4}{9} \Rightarrow du = \frac{4}{9}x^3\, dx \Rightarrow \frac{1}{4}\, du = \frac{x^3}{9}\, dx;\right.$
$\left.x = 0 \Rightarrow u = 1, x = 2 \Rightarrow u = \frac{25}{9}\right]$
$\rightarrow S = 2\pi\int_1^{25/9} u^{1/2} \cdot \frac{1}{4}\, du = \frac{\pi}{2}\left[\frac{2}{3}u^{3/2}\right]_1^{25/9}$
$= \frac{\pi}{3}\left(\frac{125}{27} - 1\right) = \frac{\pi}{3}\left(\frac{125-27}{27}\right) = \frac{98\pi}{81}$



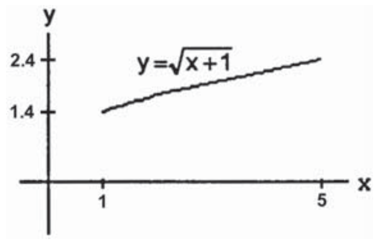
14. $\frac{dy}{dx} = \frac{1}{2}x^{-1/2} \Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{1}{4x}$
$\Rightarrow S = \int_{3/4}^{15/4} 2\pi\sqrt{x}\sqrt{1 + \frac{1}{4x}}\, dx$
$= 2\pi\int_{3/4}^{15/4}\sqrt{x + \frac{1}{4}}\, dx = 2\pi\left[\frac{2}{3}\left(x + \frac{1}{4}\right)^{3/2}\right]_{3/4}^{15/4}$
$= \frac{4\pi}{3}\left[\left(\frac{15}{4} + \frac{1}{4}\right)^{3/2} - \left(\frac{3}{4} + \frac{1}{4}\right)^{3/2}\right] = \frac{4\pi}{3}\left[\left(\frac{4}{2}\right)^3 - 1\right]$
$= \frac{4\pi}{3}(8 - 1) = \frac{28\pi}{3}$



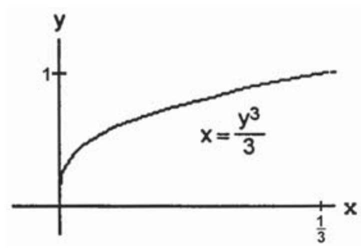
15. $\frac{dy}{dx} = \frac{1}{2}\frac{(2 - 2x)}{\sqrt{2x - x^2}} = \frac{1 - x}{\sqrt{2x - x^2}} \Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{(1 - x)^2}{2x - x^2}$
$\Rightarrow S = \int_{0.5}^{1.5} 2\pi\sqrt{2x - x^2}\sqrt{1 + \frac{(1 - x)^2}{2x - x^2}}\, dx$
$= 2\pi\int_{0.5}^{1.5}\sqrt{2x - x^2}\,\frac{\sqrt{2x - x^2 + 1 - 2x + x^2}}{\sqrt{2x - x^2}}\, dx$
$= 2\pi\int_{0.5}^{1.5} dx = 2\pi[x]_{0.5}^{1.5} = 2\pi$



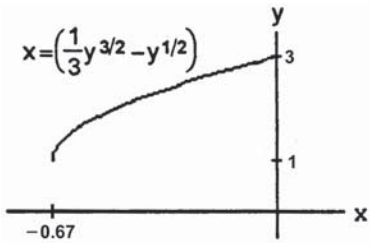
16. $\frac{dy}{dx} = \frac{1}{2\sqrt{x + 1}} \Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{1}{4(x + 1)}$
$\Rightarrow S = \int_1^5 2\pi\sqrt{x + 1}\sqrt{1 + \frac{1}{4(x + 1)}}\, dx$
$= 2\pi\int_1^5 \sqrt{(x + 1) + \frac{1}{4}}\, dx = 2\pi\int_1^5\sqrt{x + \frac{5}{4}}\, dx$
$= 2\pi\left[\frac{2}{3}\left(x + \frac{5}{4}\right)^{3/2}\right]_1^5 = \frac{4\pi}{3}\left[\left(5 + \frac{5}{4}\right)^{3/2} - \left(1 + \frac{5}{4}\right)^{3/2}\right]$
$= \frac{4\pi}{3}\left[\left(\frac{25}{4}\right)^{3/2} - \left(\frac{9}{4}\right)^{3/2}\right] = \frac{4\pi}{3}\left(\frac{5^3}{2^3} - \frac{3^3}{2^3}\right)$
$= \frac{\pi}{6}(125 - 27) = \frac{98\pi}{6} = \frac{49\pi}{3}$



17. $\frac{dx}{dy} = y^2 \Rightarrow \left(\frac{dx}{dy}\right)^2 = y^4 \Rightarrow S = \int_0^1 \frac{2\pi y^3}{3}\sqrt{1 + y^4}\, dy$;
$\left[u = 1 + y^4 \Rightarrow du = 4y^3\, dy \Rightarrow \frac{1}{4}\, du = y^3\, dy;\ y = 0\right.$
$\left.\Rightarrow u = 1, y = 1 \Rightarrow u = 2\right] \rightarrow S = \int_1^2 2\pi\left(\frac{1}{3}\right)u^{1/2}\left(\frac{1}{4}\, du\right)$
$= \frac{\pi}{6}\int_1^2 u^{1/2}\, du = \frac{\pi}{6}\left[\frac{2}{3}u^{3/2}\right]_1^2 = \frac{\pi}{9}\left(\sqrt{8} - 1\right)$



18. $x = \left(\frac{1}{3}y^{3/2} - y^{1/2}\right) \le 0$, when $1 \le y \le 3$. To get positive area, we take $x = -\left(\frac{1}{3}y^{3/2} - y^{1/2}\right)$



$\Rightarrow \frac{dx}{dy} = -\frac{1}{2}\left(y^{1/2} - y^{-1/2}\right) \Rightarrow \left(\frac{dx}{dy}\right)^2 = \frac{1}{4}\left(y - 2 + y^{-1}\right)$

$\Rightarrow S = -\int_1^3 2\pi\left(\frac{1}{3}y^{3/2} - y^{1/2}\right)\sqrt{1 + \frac{1}{4}\left(y - 2 + y^{-1}\right)}\,dy$

$= -2\pi\int_1^3 \left(\frac{1}{3}y^{3/2} - y^{1/2}\right)\sqrt{\frac{1}{4}\left(y + 2 + y^{-1}\right)}\,dy$

$= -2\pi\int_1^3 \left(\frac{1}{3}y^{3/2} - y^{1/2}\right)\frac{\sqrt{\left(y^{1/2}+y^{-1/2}\right)^2}}{2}\,dy = -\pi\int_1^3 y^{1/2}\left(\frac{1}{3}y - 1\right)\left(y^{1/2} + \frac{1}{y^{1/2}}\right)dy = -\pi\int_1^3 \left(\frac{1}{3}y - 1\right)(y+1)\,dy$

$= -\pi\int_1^3 \left(\frac{1}{3}y^2 - \frac{2}{3}y - 1\right)dy = -\pi\left[\frac{y^3}{9} - \frac{y^2}{3} - y\right]_1^3 = -\pi\left[\left(\frac{27}{9} - \frac{9}{3} - 3\right) - \left(\frac{1}{9} - \frac{1}{3} - 1\right)\right] = -\pi\left(-3 - \frac{1}{9} + \frac{1}{3} + 1\right)$

$= -\frac{\pi}{9}(-18 - 1 + 3) = \frac{16\pi}{9}$

19. $\frac{dx}{dy} = \frac{-1}{\sqrt{4-y}} \Rightarrow \left(\frac{dx}{dy}\right)^2 = \frac{1}{4-y} \Rightarrow S = \int_0^{15/4} 2\pi \cdot 2\sqrt{4-y}\,\sqrt{1 + \frac{1}{4-y}}\,dy = 4\pi\int_0^{15/4}\sqrt{(4-y)+1}\,dy$

$= 4\pi\int_0^{15/4}\sqrt{5-y}\,dy = -4\pi\left[\frac{2}{3}(5-y)^{3/2}\right]_0^{15/4} = -\frac{8\pi}{3}\left[\left(5 - \frac{15}{4}\right)^{3/2} - 5^{3/2}\right] = -\frac{8\pi}{3}\left[\left(\frac{5}{4}\right)^{3/2} - 5^{3/2}\right]$

$= \frac{8\pi}{3}\left(5\sqrt{5} - \frac{5\sqrt{5}}{8}\right) = \frac{8\pi}{3}\left(\frac{40\sqrt{5} - 5\sqrt{5}}{8}\right) = \frac{35\pi\sqrt{5}}{3}$

20. $\frac{dx}{dy} = \frac{1}{\sqrt{2y-1}} \Rightarrow \left(\frac{dx}{dy}\right)^2 = \frac{1}{2y-1} \Rightarrow S = \int_{5/8}^1 2\pi\sqrt{2y-1}\,\sqrt{1 + \frac{1}{2y-1}}\,dy = 2\pi\int_{5/8}^1 \sqrt{(2y-1)+1}\,dy$

$= 2\pi\int_{5/8}^1 \sqrt{2}\,y^{1/2}\,dy = 2\pi\sqrt{2}\left[\frac{2}{3}y^{3/2}\right]_{5/8}^1 = \frac{4\pi\sqrt{2}}{3}\left[1^{3/2} - \left(\frac{5}{8}\right)^{3/2}\right] = \frac{4\pi\sqrt{2}}{3}\left(1 - \frac{5\sqrt{5}}{8\sqrt{8}}\right)$

$= \frac{4\pi\sqrt{2}}{3}\left(\frac{8\cdot 2\sqrt{2} - 5\sqrt{5}}{8\cdot 2\sqrt{2}}\right) = \frac{\pi}{12}\left(16\sqrt{2} - 5\sqrt{5}\right)$

21. $ds = \sqrt{dx^2 + dy^2} = \sqrt{\left(y^3 - \frac{1}{4y^3}\right)^2 + 1}\,dy = \sqrt{\left(y^6 - \frac{1}{2} + \frac{1}{16y^6}\right) + 1}\,dy = \sqrt{\left(y^6 + \frac{1}{2} + \frac{1}{16y^6}\right)}\,dy$

$= \sqrt{\left(y^3 + \frac{1}{4y^3}\right)^2}\,dy = \left(y^3 + \frac{1}{4y^3}\right)dy$; $S = \int_1^2 2\pi y\,ds = 2\pi\int_1^2 y\left(y^3 + \frac{1}{4y^3}\right)dy = 2\pi\int_1^2 \left(y^4 + \frac{1}{4}y^{-2}\right)dy$

$= 2\pi\left[\frac{y^5}{5} - \frac{1}{4}y^{-1}\right]_1^2 = 2\pi\left[\left(\frac{32}{5} - \frac{1}{8}\right) - \left(\frac{1}{5} - \frac{1}{4}\right)\right] = 2\pi\left(\frac{31}{5} + \frac{1}{8}\right) = \frac{2\pi}{40}(8\cdot 31 + 5) = \frac{253\pi}{20}$

22. $y = \frac{1}{3}\left(x^2 + 2\right)^{3/2} \Rightarrow dy = x\sqrt{x^2 + 2}\,dx \Rightarrow ds = \sqrt{1 + \left(2x^2 + x^4\right)}\,dx \Rightarrow S = 2\pi\int_0^{\sqrt{2}} x\sqrt{1 + 2x^2 + x^4}\,dx$

$= 2\pi\int_0^{\sqrt{2}} x\sqrt{\left(x^2+1\right)^2}\,dx = 2\pi\int_0^{\sqrt{2}} x\left(x^2+1\right)dx = 2\pi\int_0^{\sqrt{2}}\left(x^3 + x\right)dx = 2\pi\left[\frac{x^4}{4} + \frac{x^2}{2}\right]_0^{\sqrt{2}} = 2\pi\left(\frac{4}{4} + \frac{2}{2}\right) = 4\pi$

23. $y = \sqrt{a^2 - x^2} \Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(a^2 - x^2\right)^{-1/2}(-2x) = \frac{-x}{\sqrt{a^2 - x^2}} \Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{x^2}{\left(a^2 - x^2\right)}$

$\Rightarrow S = 2\pi\int_{-a}^a \sqrt{a^2 - x^2}\,\sqrt{1 + \frac{x^2}{\left(a^2 - x^2\right)}}\,dx = 2\pi\int_{-a}^a \sqrt{\left(a^2 - x^2\right) + x^2}\,dx = 2\pi\int_{-a}^a a\,dx = 2\pi a[x]_{-a}^a$

$= 2\pi a[a - (-a)] = (2\pi a)(2a) = 4\pi a^2$

24. $y = \frac{r}{h}x \Rightarrow \frac{dy}{dx} = \frac{r}{h} \Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{r^2}{h^2} \Rightarrow S = 2\pi\int_0^h \frac{r}{h}x\sqrt{1 + \frac{r^2}{h^2}}\,dx = 2\pi\int_0^h \frac{r}{h}x\sqrt{\frac{h^2 + r^2}{h^2}}\,dx$

$= \frac{2\pi r}{h}\sqrt{\frac{h^2 + r^2}{h^2}}\int_0^h x\,dx = \frac{2\pi r}{h^2}\sqrt{h^2 + r^2}\left[\frac{x^2}{2}\right]_0^h = \frac{2\pi r}{h^2}\sqrt{h^2 + r^2}\left(\frac{h^2}{2}\right) = \pi r\sqrt{h^2 + r^2}$

25. $y = \cos x \Rightarrow \frac{dy}{dx} = -\sin x \Rightarrow \left(\frac{dy}{dx}\right)^2 = \sin^2 x \Rightarrow S = 2\pi\int_{-\pi/2}^{\pi/2} (\cos x)\sqrt{1 + \sin^2 x}\,dx$