## Finding Surface Areas

**9.** Find the lateral (side) surface area of the cone generated by revolving the line segment $y = x/2, 0 \le x \le 4$, about the $x$-axis. Check your answer with the geometry formula

$$\text{Lateral surface area} = \frac{1}{2} \times \text{base circumference} \times \text{slant height}.$$

**10.** Find the lateral surface area of the cone generated by revolving the line segment $y = x/2, 0 \le x \le 4$ about the $y$-axis. Check your answer with the geometry formula

$$\text{Lateral surface area} = \frac{1}{2} \times \text{base circumference} \times \text{slant height}.$$

**11.** Find the surface area of the cone frustum generated by revolving the line segment $y = (x/2) + (1/2), 1 \le x \le 3$, about the $x$-axis. Check your result with the geometry formula

$$\text{Frustum surface area} = \pi(r_1 + r_2) \times \text{slant height}.$$

**12.** Find the surface area of the cone frustum generated by revolving the line segment $y = (x/2) + (1/2), 1 \le x \le 3$, about the $y$-axis. Check your result with the geometry formula

$$\text{Frustum surface area} = \pi(r_1 + r_2) \times \text{slant height}.$$

Find the areas of the surfaces generated by revolving the curves in Exercises 13–22 about the indicated axes. If you have a grapher, you may want to graph these curves to see what they look like.

**13.** $y = x^3/9, \quad 0 \le x \le 2; \quad x$-axis

**14.** $y = \sqrt{x}, \quad 3/4 \le x \le 15/4; \quad x$-axis

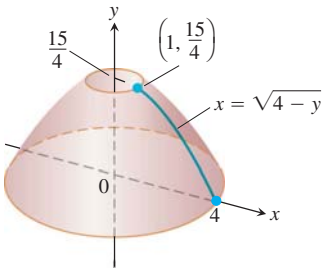
**15.** $y = \sqrt{2x - x^2}, \quad 0.5 \le x \le 1.5; \quad x$-axis

**16.** $y = \sqrt{x + 1}, \quad 1 \le x \le 5; \quad x$-axis

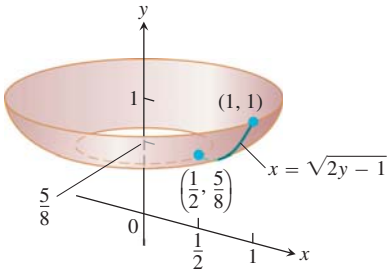
**17.** $x = y^3/3, \quad 0 \le y \le 1; \quad y$-axis

**18.** $x = (1/3)y^{3/2} - y^{1/2}, \quad 1 \le y \le 3; \quad y$-axis

**19.** $x = 2\sqrt{4 - y}, \quad 0 \le y \le 15/4; \quad y$-axis




**20.** $x = \sqrt{2y - 1}, \quad 5/8 \le y \le 1; \quad y$-axis




**21.** $x = (y^4/4) + 1/(8y^2), \quad 1 \le y \le 2; \quad x$-axis (*Hint:* Express $ds = \sqrt{dx^2 + dy^2}$ in terms of $dy$, and evaluate the integral $S = \int 2\pi y \, ds$ with appropriate limits.)

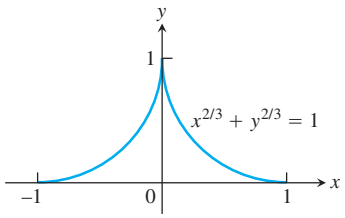
**22.** $y = (1/3)(x^2 + 2)^{3/2}, \quad 0 \le x \le \sqrt{2}; \quad y$-axis (*Hint:* Express $ds = \sqrt{dx^2 + dy^2}$ in terms of $dx$, and evaluate the integral $S = \int 2\pi x \, ds$ with appropriate limits.)

**23. Testing the new definition** Show that the surface area of a sphere of radius $a$ is still $4\pi a^2$ by using Equation (3) to find the area of the surface generated by revolving the curve $y = \sqrt{a^2 - x^2}, -a \le x \le a$, about the $x$-axis.

**24. Testing the new definition** The lateral (side) surface area of a cone of height $h$ and base radius $r$ should be $\pi r\sqrt{r^2 + h^2}$, the semiperimeter of the base times the slant height. Show that this is still the case by finding the area of the surface generated by revolving the line segment $y = (r/h)x, 0 \le x \le h$, about the $x$-axis.

**25.** Write an integral for the area of the surface generated by revolving the curve $y = \cos x, -\pi/2 \le x \le \pi/2$, about the $x$-axis. In Section 8.5 we will see how to evaluate such integrals.

**26. The surface of an astroid** Find the area of the surface generated by revolving about the $x$-axis the portion of the astroid $x^{2/3} + y^{2/3} = 1$ shown here. (*Hint:* Revolve the first-quadrant portion $y = (1 - x^{2/3})^{3/2}, 0 \le x \le 1$, about the $x$-axis and double your result.)




T **27. Enameling woks** Your company decided to put out a deluxe version of the successful wok you designed in Section 6.1, Exercise 55. The plan is to coat it inside with white enamel and outside with blue enamel. Each enamel will be sprayed on 0.5 mm thick before baking. (See diagram here.) Your manufacturing department wants to know how much enamel to have on hand for a production run of 5000 woks. What do you tell them? (Neglect waste and unused material and give your answer in liters. Remember that $1 \text{ cm}^3 = 1 \text{ mL}$, so $1 \text{ L} = 1000 \text{ cm}^3$.)