(c) $\int [(x^2-1)(x+1)]^{-2/3}\,dx = \int (x+1)^{-2}\left(\frac{x-1}{x+1}\right)^{-2/3} dx;$

$\begin{bmatrix} u = \tan^{-1} x \\ x = \tan u \\ dx = \frac{du}{\cos^2 u} \end{bmatrix} \to \int \frac{1}{(\tan u+1)^2}\left(\frac{\tan u-1}{\tan u+1}\right)^{-2/3}\left(\frac{du}{\cos^2 u}\right) = \int \frac{1}{(\sin u+\cos u)^2}\left(\frac{\sin u-\cos u}{\sin u+\cos u}\right)^{-2/3} du;$

$\begin{bmatrix} \sin u + \cos u = \sin u + \sin\left(\frac{\pi}{2}-u\right) = 2\sin\frac{\pi}{4}\cos\left(u-\frac{\pi}{4}\right) \\ \sin u - \cos u = \sin u - \sin\left(\frac{\pi}{2}-u\right) = 2\cos\frac{\pi}{4}\sin\left(u-\frac{\pi}{4}\right) \end{bmatrix} \to \int \frac{1}{2\cos^2\left(u-\frac{\pi}{4}\right)}\left[\frac{\sin\left(u-\frac{\pi}{4}\right)}{\cos\left(u-\frac{\pi}{4}\right)}\right]^{-2/3} du$

$= \frac{1}{2}\int \tan^{-2/3}\left(u-\frac{\pi}{4}\right)\sec^2\left(u-\frac{\pi}{4}\right) du = \frac{3}{2}\tan^{1/3}\left(u-\frac{\pi}{4}\right) + C = \frac{3}{2}\left[\frac{\tan u - \tan\frac{\pi}{4}}{1+\tan u\tan\frac{\pi}{4}}\right]^{1/3} + C$

$= \frac{3}{2}\left(\frac{x-1}{x+1}\right)^{1/3} + C$

(d) $u = \tan^{-1}\sqrt{x} \Rightarrow \tan u = \sqrt{x} \Rightarrow \tan^2 u = x \Rightarrow dx = 2\tan u\left(\frac{1}{\cos^2 u}\right) du = \frac{2\sin u}{\cos^3 u}\,du = -\frac{2d(\cos u)}{\cos^3 u};$

$x - 1 = \tan^2 u - 1 = \frac{\sin^2 u - \cos^2 u}{\cos^2 u} = \frac{1-2\cos^2 u}{\cos^2 u}$; $x + 1 = \tan^2 u + 1 = \frac{\cos^2 u + \sin^2 u}{\cos^2 u} = \frac{1}{\cos^2 u}$;

$\int (x-1)^{-2/3}(x+1)^{-4/3}\,dx = \int \frac{(1-2\cos^2 u)^{-2/3}}{(\cos^2 u)^{-2/3}} \cdot \frac{1}{(\cos^2 u)^{-4/3}} \cdot \frac{-2d(\cos u)}{\cos^3 u}$

$= \int (1-2\cos^2 u)^{-2/3} \cdot (-2) \cdot \cos u \cdot d(\cos u) = \frac{1}{2}\int (1-2\cos^2 u)^{-2/3} \cdot d(1-2\cos^2 u)$

$= \frac{3}{2}(1-2\cos^2 u)^{1/3} + C = \frac{3}{2}\left[\frac{\left(\frac{1-2\cos^2 u}{\cos^2 u}\right)}{\left(\frac{1}{\cos^2 u}\right)}\right]^{1/3} + C = \frac{3}{2}\left(\frac{x-1}{x+1}\right)^{1/3} + C$

(e) $u = \tan^{-1}\left(\frac{x-1}{2}\right) \Rightarrow \frac{x-1}{2} = \tan u \Rightarrow x + 1 = 2(\tan u + 1) \Rightarrow dx = \frac{2\,du}{\cos^2 u} = 2d(\tan u);$

$\int (x-1)^{-2/3}(x+1)^{-4/3}\,dx = \int (\tan u)^{-2/3}(\tan u+1)^{-4/3} \cdot 2^{-2} \cdot 2 \cdot d(\tan u)$

$= \frac{1}{2}\int \left(1-\frac{1}{\tan u+1}\right)^{-2/3} d\left(1-\frac{1}{\tan u+1}\right) = \frac{3}{2}\left(1-\frac{1}{\tan u+1}\right)^{1/3} + C = \frac{3}{2}\left(1-\frac{2}{x+1}\right)^{1/3} + C$

$= \frac{3}{2}\left(\frac{x-1}{x+1}\right)^{1/3} + C$

(f) $\begin{bmatrix} u = \cos^{-1} x \\ x = \cos u \\ dx = -\sin u\,du \end{bmatrix} \to -\int \frac{\sin u\,du}{\sqrt[3]{(\cos^2 u - 1)^2(\cos u+1)^2}} = -\int \frac{\sin u\,du}{(\sin^{4/3} u)\left(2^{2/3}\cos\frac{u}{2}\right)^{4/3}}$

$= -\int \frac{du}{(\sin u)^{1/3}\left(2^{2/3}\cos\frac{u}{2}\right)^{4/3}} = -\int \frac{du}{2\left(\sin\frac{u}{2}\right)^{1/3}\left(\cos\frac{u}{2}\right)^{5/3}} = -\frac{1}{2}\int \left(\frac{\cos\frac{u}{2}}{\sin\frac{u}{2}}\right)^{1/3}\frac{du}{\left(\cos^2\frac{u}{2}\right)}$

$= -\int \tan^{-1/3}\left(\frac{u}{2}\right) d\left(\tan\frac{u}{2}\right) = -\frac{3}{2}\tan^{2/3}\frac{u}{2} + C = \frac{3}{2}\left(-\tan^2\frac{u}{2}\right)^{1/3} + C = \frac{3}{2}\left(\frac{\cos u - 1}{\cos u + 1}\right)^{1/3} + C$

$= \frac{3}{2}\left(\frac{x-1}{x+1}\right)^{1/3} + C$

(g) $\int [(x^2-1)(x+1)]^{-2/3}\,dx;\ \begin{bmatrix} u = \cosh^{-1} x \\ x = \cosh u \\ dx = \sinh u \end{bmatrix} \to \int \frac{\sinh u\,du}{\sqrt[3]{(\cosh^2 u-1)^2(\cosh u+1)^2}}$

$= \int \frac{\sinh u\,du}{\sqrt[3]{(\sinh^4 u)(\cosh u+1)^2}} = \int \frac{du}{\sqrt[3]{(\sinh u)\left(4\cosh^4\frac{u}{2}\right)}} = \frac{1}{2}\int \frac{du}{\sqrt[3]{\sinh\left(\frac{u}{2}\right)\cosh^5\left(\frac{u}{2}\right)}}$

$= \int \left(\tanh\frac{u}{2}\right)^{-1/3} d\left(\tanh\frac{u}{2}\right) = \frac{3}{2}\left(\tanh\frac{u}{2}\right)^{2/3} + C = \frac{3}{2}\left(\frac{\cosh u - 1}{\cosh u + 1}\right)^{1/3} + C = \frac{3}{2}\left(\frac{x-1}{x+1}\right)^{1/3} + C$

8.2 INTEGRATION BY PARTS

1. $u = x$, $du = dx$; $dv = \sin\frac{x}{2}\,dx$, $v = -2\cos\frac{x}{2}$;

$\int x\sin\frac{x}{2}\,dx = -2x\cos\frac{x}{2} - \int\left(-2\cos\frac{x}{2}\right) dx = -2x\cos\left(\frac{x}{2}\right) + 4\sin\left(\frac{x}{2}\right) + C$

2. $u = \theta$, $du = d\theta$; $dv = \cos\pi\theta\,d\theta$, $v = \frac{1}{\pi}\sin\pi\theta$;

$\int \theta\cos\pi\theta\,d\theta = \frac{\theta}{\pi}\sin\pi\theta - \int\frac{1}{\pi}\sin\pi\theta\,d\theta = \frac{\theta}{\pi}\sin\pi\theta + \frac{1}{\pi^2}\cos\pi\theta + C$

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3.

		$\cos t$
t^2	$\xrightarrow{(+)}$	$\sin t$
$2t$	$\xrightarrow{(-)}$	$-\cos t$
2	$\xrightarrow{(+)}$	$-\sin t$
0		

$$\int t^2 \cos t\, dt = t^2 \sin t + 2t \cos t - 2 \sin t + C$$

4.

		$\sin x$
x^2	$\xrightarrow{(+)}$	$-\cos x$
$2x$	$\xrightarrow{(-)}$	$-\sin x$
2	$\xrightarrow{(+)}$	$\cos x$
0		

$$\int x^2 \sin x\, dx = -x^2 \cos x + 2x \sin x + 2 \cos x + C$$

5. $u = \ln x,\ du = \frac{dx}{x};\ dv = x\, dx,\ v = \frac{x^2}{2};$
$\int_1^2 x \ln x\, dx = \left[\frac{x^2}{2} \ln x\right]_1^2 - \int_1^2 \frac{x^2}{2} \frac{dx}{x} = 2 \ln 2 - \left[\frac{x^2}{4}\right]_1^2 = 2 \ln 2 - \frac{3}{4} = \ln 4 - \frac{3}{4}$

6. $u = \ln x,\ du = \frac{dx}{x};\ dv = x^3\, dx,\ v = \frac{x^4}{4};$
$\int_1^e x^3 \ln x\, dx = \left[\frac{x^4}{4} \ln x\right]_1^e - \int_1^e \frac{x^4}{4} \frac{dx}{x} = \frac{e^4}{4} - \left[\frac{x^4}{16}\right]_1^e = \frac{3e^4+1}{16}$

7. $u = \tan^{-1} y,\ du = \frac{dy}{1+y^2};\ dv = dy,\ v = y;$
$\int \tan^{-1} y\, dy = y \tan^{-1} y - \int \frac{y\, dy}{(1+y^2)} = y \tan^{-1} y - \frac{1}{2} \ln(1+y^2) + C = y \tan^{-1} y - \ln \sqrt{1+y^2} + C$

8. $u = \sin^{-1} y,\ du = \frac{dy}{\sqrt{1-y^2}};\ dv = dy,\ v = y;$
$\int \sin^{-1} y\, dy = y \sin^{-1} y - \int \frac{y\, dy}{\sqrt{1-y^2}} = y \sin^{-1} y + \sqrt{1-y^2} + C$

9. $u = x,\ du = dx;\ dv = \sec^2 x\, dx,\ v = \tan x;$
$\int x \sec^2 x\, dx = x \tan x - \int \tan x\, dx = x \tan x + \ln|\cos x| + C$

10. $\int 4x \sec^2 2x\, dx;\ [y = 2x] \rightarrow \int y \sec^2 y\, dy = y \tan y - \int \tan y\, dy = y \tan y - \ln|\sec y| + C$
$= 2x \tan 2x - \ln|\sec 2x| + C$

11.

		e^x
x^3	$\xrightarrow{(+)}$	e^x
$3x^2$	$\xrightarrow{(-)}$	e^x
$6x$	$\xrightarrow{(+)}$	e^x
6	$\xrightarrow{(-)}$	e^x
0		

$$\int x^3 e^x\, dx = x^3 e^x - 3x^2 e^x + 6x e^x - 6e^x + C = (x^3 - 3x^2 + 6x - 6)\, e^x + C$$

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12.

		e^{-p}
p^4	$(+)$ →	$-e^{-p}$
$4p^3$	$(-)$ →	e^{-p}
$12p^2$	$(+)$ →	$-e^{-p}$
$24p$	$(-)$ →	e^{-p}
24	$(+)$ →	$-e^{-p}$
0		

$$\int p^4e^{-p}\,dp = -p^4e^{-p} - 4p^3e^{-p} - 12p^2e^{-p} - 24pe^{-p} - 24e^{-p} + C$$
$$= (-p^4 - 4p^3 - 12p^2 - 24p - 24)\,e^{-p} + C$$

13.

		e^x
$x^2 - 5x$	$(+)$ →	e^x
$2x - 5$	$(-)$ →	e^x
2	$(+)$ →	e^x
0		

$$\int (x^2 - 5x)\,e^x\,dx = (x^2 - 5x)\,e^x - (2x - 5)e^x + 2e^x + C = x^2e^x - 7xe^x + 7e^x + C$$
$$= (x^2 - 7x + 7)\,e^x + C$$

14.

		e^r
$r^2 + r + 1$	$(+)$ →	e^r
$2r + 1$	$(-)$ →	e^r
2	$(+)$ →	e^r
0		

$$\int (r^2 + r + 1)\,e^r\,dr = (r^2 + r + 1)\,e^r - (2r + 1)\,e^r + 2e^r + C$$
$$= [(r^2 + r + 1) - (2r + 1) + 2]\,e^r + C = (r^2 - r + 2)\,e^r + C$$

15.

		e^x
x^5	$(+)$ →	e^x
$5x^4$	$(-)$ →	e^x
$20x^3$	$(+)$ →	e^x
$60x^2$	$(-)$ →	e^x
$120x$	$(+)$ →	e^x
120	$(-)$ →	e^x
0		

$$\int x^5e^x\,dx = x^5e^x - 5x^4e^x + 20x^3e^x - 60x^2e^x + 120xe^x - 120e^x + C$$
$$= (x^5 - 5x^4 + 20x^3 - 60x^2 + 120x - 120)\,e^x + C$$

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16.

		e^{4t}
t^2	$\xrightarrow{(+)}$	$\frac{1}{4}e^{4t}$
$2t$	$\xrightarrow{(-)}$	$\frac{1}{16}e^{4t}$
2	$\xrightarrow{(+)}$	$\frac{1}{64}e^{4t}$
0		

$$\int t^2e^{4t}\,dt = \frac{t^2}{4}e^{4t} - \frac{2t}{16}e^{4t} + \frac{2}{64}e^{4t} + C = \frac{t^2}{4}e^{4t} - \frac{t}{8}e^{4t} + \frac{1}{32}e^{4t} + C$$
$$= \left(\frac{t^2}{4} - \frac{t}{8} + \frac{1}{32}\right)e^{4t} + C$$

17.

		$\sin 2\theta$
θ^2	$\xrightarrow{(+)}$	$-\frac{1}{2}\cos 2\theta$
2θ	$\xrightarrow{(-)}$	$-\frac{1}{4}\sin 2\theta$
2	$\xrightarrow{(+)}$	$\frac{1}{8}\cos 2\theta$
0		

$$\int_0^{\pi/2}\theta^2\sin 2\theta\,d\theta = \left[-\frac{\theta^2}{2}\cos 2\theta + \frac{\theta}{2}\sin 2\theta + \frac{1}{4}\cos 2\theta\right]_0^{\pi/2}$$
$$= \left[-\frac{\pi^2}{8}\cdot(-1) + \frac{\pi}{4}\cdot 0 + \frac{1}{4}\cdot(-1)\right] - \left[0 + 0 + \frac{1}{4}\cdot 1\right] = \frac{\pi^2}{8} - \frac{1}{2} = \frac{\pi^2-4}{8}$$

18.

		$\cos 2x$
x^3	$\xrightarrow{(+)}$	$\frac{1}{2}\sin 2x$
$3x^2$	$\xrightarrow{(-)}$	$-\frac{1}{4}\cos 2x$
$6x$	$\xrightarrow{(+)}$	$-\frac{1}{8}\sin 2x$
6	$\xrightarrow{(-)}$	$\frac{1}{16}\cos 2x$
0		

$$\int_0^{\pi/2} x^3\cos 2x\,dx = \left[\frac{x^3}{2}\sin 2x + \frac{3x^2}{4}\cos 2x - \frac{3x}{4}\sin 2x - \frac{3}{8}\cos 2x\right]_0^{\pi/2}$$
$$= \left[\frac{\pi^3}{16}\cdot 0 + \frac{3\pi^2}{16}\cdot(-1) - \frac{3\pi}{8}\cdot 0 - \frac{3}{8}\cdot(-1)\right] - \left[0 + 0 - 0 - \frac{3}{8}\cdot 1\right] = -\frac{3\pi^2}{16} + \frac{3}{4} = \frac{3(4-\pi^2)}{16}$$

19. $u = \sec^{-1} t$, $du = \frac{dt}{t\sqrt{t^2-1}}$; $dv = t\,dt$, $v = \frac{t^2}{2}$;

$$\int_{2/\sqrt{3}}^{2} t\sec^{-1} t\,dt = \left[\frac{t^2}{2}\sec^{-1} t\right]_{2/\sqrt{3}}^{2} - \int_{2/\sqrt{3}}^{2}\left(\frac{t^2}{2}\right)\frac{dt}{t\sqrt{t^2-1}} = \left(2\cdot\frac{\pi}{3} - \frac{2}{3}\cdot\frac{\pi}{6}\right) - \int_{2/\sqrt{3}}^{2}\frac{t\,dt}{2\sqrt{t^2-1}}$$
$$= \frac{5\pi}{9} - \left[\frac{1}{2}\sqrt{t^2-1}\right]_{2/\sqrt{3}}^{2} = \frac{5\pi}{9} - \frac{1}{2}\left(\sqrt{3} - \sqrt{\frac{4}{3}-1}\right) = \frac{5\pi}{9} - \frac{1}{2}\left(\sqrt{3} - \frac{\sqrt{3}}{3}\right) = \frac{5\pi}{9} - \frac{\sqrt{3}}{3} = \frac{5\pi - 3\sqrt{3}}{9}$$

20. $u = \sin^{-1}(x^2)$, $du = \frac{2x\,dx}{\sqrt{1-x^4}}$; $dv = 2x\,dx$, $v = x^2$;

$$\int_0^{1/\sqrt{2}} 2x\sin^{-1}(x^2)\,dx = \left[x^2\sin^{-1}(x^2)\right]_0^{1/\sqrt{2}} - \int_0^{1/\sqrt{2}} x^2\cdot\frac{2x\,dx}{\sqrt{1-x^4}} = \left(\frac{1}{2}\right)\left(\frac{\pi}{6}\right) + \int_0^{1/\sqrt{2}}\frac{d(1-x^4)}{2\sqrt{1-x^4}}$$
$$= \frac{\pi}{12} + \left[\sqrt{1-x^4}\right]_0^{1/\sqrt{2}} = \frac{\pi}{12} + \sqrt{\frac{3}{4}} - 1 = \frac{\pi+6\sqrt{3}-12}{12}$$

21. $I = \int e^\theta\sin\theta\,d\theta$; $[u = \sin\theta,\ du = \cos\theta\,d\theta;\ dv = e^\theta\,d\theta,\ v = e^\theta] \Rightarrow I = e^\theta\sin\theta - \int e^\theta\cos\theta\,d\theta$;
$[u = \cos\theta,\ du = -\sin\theta\,d\theta;\ dv = e^\theta\,d\theta,\ v = e^\theta] \Rightarrow I = e^\theta\sin\theta - \left(e^\theta\cos\theta + \int e^\theta\sin\theta\,d\theta\right)$
$= e^\theta\sin\theta - e^\theta\cos\theta - I + C' \Rightarrow 2I = (e^\theta\sin\theta - e^\theta\cos\theta) + C' \Rightarrow I = \frac{1}{2}(e^\theta\sin\theta - e^\theta\cos\theta) + C$, where $C = \frac{C'}{2}$ is another arbitrary constant

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22. $I = \int e^{-y} \cos y \, dy$; $[u = \cos y,\ du = -\sin y \, dy;\ dv = e^{-y} dy,\ v = -e^{-y}]$
$\Rightarrow I = -e^{-y} \cos y - \int (-e^{-y})(-\sin y)\, dy = -e^{-y} \cos y - \int e^{-y} \sin y \, dy$; $[u = \sin y,\ du = \cos y \, dy;$
$dv = e^{-y} dy,\ v = -e^{-y}] \Rightarrow I = -e^{-y} \cos y - \left(-e^{-y} \sin y - \int (-e^{y}) \cos y \, dy\right) = -e^{-y} \cos y + e^{-y} \sin y - I + C'$
$\Rightarrow 2I = e^{-y}(\sin y - \cos y) + C' \Rightarrow I = \frac{1}{2}(e^{-y} \sin y - e^{-y} \cos y) + C$, where $C = \frac{C'}{2}$ is another arbitrary constant

23. $I = \int e^{2x} \cos 3x \, dx$; $\left[u = \cos 3x;\ du = -3 \sin 3x \, dx,\ dv = e^{2x} dx;\ v = \frac{1}{2} e^{2x}\right]$
$\Rightarrow I = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{2} \int e^{2x} \sin 3x \, dx$; $\left[u = \sin 3x,\ du = 3 \cos 3x,\ dv = e^{2x} dx;\ v = \frac{1}{2} e^{2x}\right]$
$\Rightarrow I = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{2}\left(\frac{1}{2} e^{2x} \sin 3x - \frac{3}{2} \int e^{2x} \cos 3x \, dx\right) = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{4} e^{2x} \sin 3x - \frac{9}{4} I + C'$
$\Rightarrow \frac{13}{4} I = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{4} e^{2x} \sin 3x + C' \Rightarrow \frac{e^{2x}}{13}(3 \sin 3x + 2 \cos 3x) + C$, where $C = \frac{4}{13} C'$

24. $\int e^{-2x} \sin 2x \, dx$; $[y = 2x] \rightarrow \frac{1}{2} \int e^{-y} \sin y \, dy = I$; $[u = \sin y,\ du = \cos y \, dy;\ dv = e^{-y} dy,\ v = -e^{-y}]$
$\Rightarrow I = \frac{1}{2}\left(-e^{-y} \sin y + \int e^{-y} \cos y \, dy\right)$ $[u = \cos y,\ du = -\sin y;\ dv = e^{-y} dy,\ v = -e^{-y}]$
$\Rightarrow I = -\frac{1}{2} e^{-y} \sin y + \frac{1}{2}\left(-e^{-y} \cos y - \int (-e^{-y})(-\sin y)\, dy\right) = -\frac{1}{2} e^{-y}(\sin y + \cos y) - I + C'$
$\Rightarrow 2I = -\frac{1}{2} e^{-y}(\sin y + \cos y) + C' \Rightarrow I = -\frac{1}{4} e^{-y}(\sin y + \cos y) + C = -\frac{e^{-2x}}{4}(\sin 2x + \cos 2x) + C$, where
$C = \frac{C'}{2}$

25. $\int e^{\sqrt{3s+9}} \, ds$; $\begin{bmatrix} 3s + 9 = x^2 \\ ds = \frac{2}{3} x \, dx \end{bmatrix} \rightarrow \int e^x \cdot \frac{2}{3} x \, dx = \frac{2}{3} \int x e^x \, dx$; $[u = x,\ du = dx;\ dv = e^x dx,\ v = e^x]$;
$\frac{2}{3} \int x e^x \, dx = \frac{2}{3}\left(x e^x - \int e^x \, dx\right) = \frac{2}{3}(x e^x - e^x) + C = \frac{2}{3}\left(\sqrt{3s+9}\, e^{\sqrt{3s+9}} - e^{\sqrt{3s+9}}\right) + C$

26. $u = x,\ du = dx;\ dv = \sqrt{1-x}\, dx,\ v = -\frac{2}{3}\sqrt{(1-x)^3}$;
$\int_0^1 x\sqrt{1-x}\, dx = \left[-\frac{2}{3}\sqrt{(1-x)^3}\, x\right]_0^1 + \frac{2}{3}\int_0^1 \sqrt{(1-x)^3}\, dx = \frac{2}{3}\left[-\frac{2}{5}(1-x)^{5/2}\right]_0^1 = \frac{4}{15}$

27. $u = x,\ du = dx;\ dv = \tan^2 x \, dx,\ v = \int \tan^2 x \, dx = \int \frac{\sin^2 x}{\cos^2 x}\, dx = \int \frac{1 - \cos^2 x}{\cos^2 x}\, dx = \int \frac{dx}{\cos^2 x} - \int dx$
$= \tan x - x$; $\int_0^{\pi/3} x \tan^2 x \, dx = [x(\tan x - x)]_0^{\pi/3} - \int_0^{\pi/3} (\tan x - x)\, dx = \frac{\pi}{3}\left(\sqrt{3} - \frac{\pi}{3}\right) + \left[\ln|\cos x| + \frac{x^2}{2}\right]_0^{\pi/3}$
$= \frac{\pi}{3}\left(\sqrt{3} - \frac{\pi}{3}\right) + \ln\frac{1}{2} + \frac{\pi^2}{18} = \frac{\pi\sqrt{3}}{3} - \ln 2 - \frac{\pi^2}{18}$

28. $u = \ln(x + x^2),\ du = \frac{(2x+1)\, dx}{x + x^2}$; $dv = dx,\ v = x$; $\int \ln(x + x^2)\, dx = x \ln(x + x^2) - \int \frac{2x+1}{x(x+1)} \cdot x \, dx$
$= x \ln(x + x^2) - \int \frac{(2x+1)\, dx}{x+1} = x \ln(x + x^2) - \int \frac{2(x+1) - 1}{x+1}\, dx = x \ln(x + x^2) - 2x + \ln|x + 1| + C$

29. $\int \sin(\ln x)\, dx$; $\begin{bmatrix} u = \ln x \\ du = \frac{1}{x} dx \\ dx = e^u du \end{bmatrix} \rightarrow \int (\sin u) e^u \, du$. From Exercise 21, $\int (\sin u) e^u \, du = e^u\left(\frac{\sin u - \cos u}{2}\right) + C$
$= \frac{1}{2}[-x \cos(\ln x) + x \sin(\ln x)] + C$

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30. $\int z(\ln z)^2\,dz;\ \begin{bmatrix} u = \ln z \\ du = \frac{1}{z}\,dz \\ dz = e^u\,du \end{bmatrix} \rightarrow \int e^u \cdot u^2 \cdot e^u\,du = \int e^{2u} \cdot u^2\,du;$

$$\begin{array}{lcl} & & e^{2u} \\ u^2 & \xrightarrow{(+)} & \frac{1}{2}e^{2u} \\ 2u & \xrightarrow{(-)} & \frac{1}{4}e^{2u} \\ 2 & \xrightarrow{(+)} & \frac{1}{8}e^{2u} \\ 0 & & \end{array}$$

$$\int u^2e^{2u}\,du = \frac{u^2}{2}e^{2u} - \frac{u}{2}e^{2u} + \frac{1}{4}e^{2u} + C = \frac{e^{2u}}{4}[2u^2 - 2u + 1] + C$$
$$= \frac{z^2}{4}[2(\ln z)^2 - 2\ln z + 1] + C$$

31. (a) $u = x,\ du = dx;\ dv = \sin x\,dx,\ v = -\cos x;$

$S_1 = \int_0^{\pi} x\sin x\,dx = [-x\cos x]_0^{\pi} + \int_0^{\pi}\cos x\,dx = \pi + [\sin x]_0^{\pi} = \pi$

(b) $S_2 = -\int_{\pi}^{2\pi} x\sin x\,dx = -\left[[-x\cos x]_{\pi}^{2\pi} + \int_{\pi}^{2\pi}\cos x\,dx\right] = -\left[-3\pi + [\sin x]_{\pi}^{2\pi}\right] = 3\pi$

(c) $S_3 = \int_{2\pi}^{3\pi} x\sin x\,dx = [-x\cos x]_{2\pi}^{3\pi} + \int_{2\pi}^{3\pi}\cos x\,dx = 5\pi + [\sin x]_{2\pi}^{3\pi} = 5\pi$

(d) $S_{n+1} = (-1)^{n+1}\int_{n\pi}^{(n+1)\pi} x\sin x\,dx = (-1)^{n+1}\left[[-x\cos x]_{n\pi}^{(n+1)\pi} + [\sin x]_{n\pi}^{(n+1)\pi}\right]$

$= (-1)^{n+1}\left[-(n+1)\pi(-1)^n + n\pi(-1)^{n+1}\right] + 0 = (2n+1)\pi$

32. (a) $u = x,\ du = dx;\ dv = \cos x\,dx,\ v = \sin x;$

$S_1 = -\int_{\pi/2}^{3\pi/2} x\cos x\,dx = -\left[[x\sin x]_{\pi/2}^{3\pi/2} - \int_{\pi/2}^{3\pi/2}\sin x\,dx\right] = -\left(-\frac{3\pi}{2} - \frac{\pi}{2}\right) - [\cos x]_{\pi/2}^{3\pi/2} = 2\pi$

(b) $S_2 = \int_{3\pi/2}^{5\pi/2} x\cos x\,dx = [x\sin x]_{3\pi/2}^{5\pi/2} - \int_{3\pi/2}^{5\pi/2}\sin x\,dx = \left[\frac{5\pi}{2} - \left(-\frac{3\pi}{2}\right)\right] - [\cos x]_{3\pi/2}^{5\pi/2} = 4\pi$

(c) $S_3 = -\int_{5\pi/2}^{7\pi/2} x\cos x\,dx = -\left[[x\sin x]_{5\pi/2}^{7\pi/2} - \int_{5\pi/2}^{7\pi/2}\sin x\,dx\right] = -\left(-\frac{7\pi}{2} - \frac{5\pi}{2}\right) - [\cos x]_{5\pi/2}^{7\pi/2} = 6\pi$

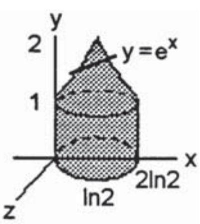
(d) $S_n = (-1)^n\int_{(2n-1)\pi/2}^{(2n+1)\pi/2} x\cos x\,dx = (-1)^n\left[[x\sin x]_{(2n-1)\pi/2}^{(2n+1)\pi/2} - {}^{n}\int_{(2n-1)\pi/2}^{(2n+1)\pi/2}\sin x\,dx\right]$

$= (-1)^n\left[\frac{(2n+1)\pi}{2}(-1)^n - \frac{(2n-1)\pi}{2}(-1)^{n-1}\right] - [\cos x]_{(2n-1)\pi/2}^{(2n+1)\pi/2} = \frac{1}{2}(2n\pi + \pi + 2n\pi - \pi) = 2n\pi$

33. $V = \int_0^{\ln 2} 2\pi(\ln 2 - x)e^x\,dx = 2\pi\ln 2\int_0^{\ln 2} e^x\,dx - 2\pi\int_0^{\ln 2} xe^x\,dx$

$= (2\pi\ln 2)\left[e^x\right]_0^{\ln 2} - 2\pi\left(\left[xe^x\right]_0^{\ln 2} - \int_0^{\ln 2} e^x\,dx\right)$

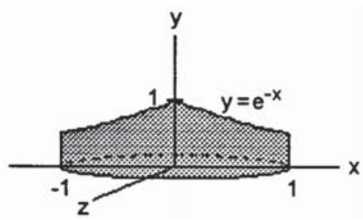
$= 2\pi\ln 2 - 2\pi\left(2\ln 2 - \left[e^x\right]_0^{\ln 2}\right) = -2\pi\ln 2 + 2\pi = 2\pi(1 - \ln 2)$



34. (a) $V = \int_0^1 2\pi xe^{-x}\,dx = 2\pi\left(\left[-xe^{-x}\right]_0^1 + \int_0^1 e^{-x}\,dx\right)$

$= 2\pi\left(-\frac{1}{e} + \left[-e^{-x}\right]_0^1\right) = 2\pi\left(-\frac{1}{e} - \frac{1}{e} + 1\right)$

$= 2\pi - \frac{4\pi}{e}$



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