

Basic Trigonometric Substitutions

Evaluate the integrals in Exercises 1–28.

1. $\int \frac{dy}{\sqrt{9 + y^2}}$

2. $\int \frac{3 dy}{\sqrt{1 + 9y^2}}$

3. $\int_{-2}^2 \frac{dx}{4 + x^2}$

4. $\int_0^2 \frac{dx}{8 + 2x^2}$

5. $\int_0^{3/2} \frac{dx}{\sqrt{9 - x^2}}$

6. $\int_0^{1/2\sqrt{2}} \frac{2 dx}{\sqrt{1 - 4x^2}}$

7. $\int \sqrt{25 - t^2} dt$

8. $\int \sqrt{1 - 9t^2} dt$

9. $\int \frac{dx}{\sqrt{4x^2 - 49}}, x > \frac{7}{2}$

10. $\int \frac{5 dx}{\sqrt{25x^2 - 9}}, x > \frac{3}{5}$

11. $\int \frac{\sqrt{y^2 - 49}}{y} dy, y > 7$

12. $\int \frac{\sqrt{y^2 - 25}}{y^3} dy, y > 5$

13. $\int \frac{dx}{x^2\sqrt{x^2 - 1}}, x > 1$

14. $\int \frac{2 dx}{x^3\sqrt{x^2 - 1}}, x > 1$

15. $\int \frac{x^3 dx}{\sqrt{x^2 + 4}}$

16. $\int \frac{dx}{x^2\sqrt{x^2 + 1}}$

17. $\int \frac{8 dw}{w^2\sqrt{4 - w^2}}$

18. $\int \frac{\sqrt{9 - w^2}}{w^2} dw$

19. $\int_0^{\sqrt{3}/2} \frac{4x^2 dx}{(1 - x^2)^{3/2}}$

20. $\int_0^1 \frac{dx}{(4 - x^2)^{3/2}}$

21. $\int \frac{dx}{(x^2 - 1)^{3/2}}, x > 1$

22. $\int \frac{x^2 dx}{(x^2 - 1)^{5/2}}, x > 1$

23. $\int \frac{(1 - x^2)^{3/2}}{x^6} dx$

24. $\int \frac{(1 - x^2)^{1/2}}{x^4} dx$

25. $\int \frac{8 dx}{(4x^2 + 1)^2}$

26. $\int \frac{6 dt}{(9t^2 + 1)^2}$

27. $\int \frac{v^2 dv}{(1 - v^2)^{5/2}}$

28. $\int \frac{(1 - r^2)^{5/2}}{r^8} dr$

In Exercises 29–36, use an appropriate substitution and then a trigonometric substitution to evaluate the integrals.

29. $\int_0^{\ln 4} \frac{e^t dt}{\sqrt{e^{2t} + 9}}$

30. $\int_{\ln(3/4)}^{\ln(4/3)} \frac{e^t dt}{(1 + e^{2t})^{3/2}}$

31. $\int_{1/12}^{1/4} \frac{2 dt}{\sqrt{t} + 4t\sqrt{t}}$

32. $\int_1^e \frac{dy}{y\sqrt{1 + (\ln y)^2}}$

33. $\int \frac{dx}{x\sqrt{x^2 - 1}}$

34. $\int \frac{dx}{1 + x^2}$

35. $\int \frac{x dx}{\sqrt{x^2 - 1}}$

36. $\int \frac{dx}{\sqrt{1 - x^2}}$