

Expanding Quotients into Partial Fractions

Expand the quotients in Exercises 1–8 by partial fractions.

1. $\frac{5x - 13}{(x - 3)(x - 2)}$

2. $\frac{5x - 7}{x^2 - 3x + 2}$

3. $\frac{x + 4}{(x + 1)^2}$

4. $\frac{2x + 2}{x^2 - 2x + 1}$

5. $\frac{z + 1}{z^2(z - 1)}$

6. $\frac{z}{z^3 - z^2 - 6z}$

7. $\frac{t^2 + 8}{t^2 - 5t + 6}$

8. $\frac{t^4 + 9}{t^4 + 9t^2}$

Nonrepeated Linear Factors

In Exercises 9–16, express the integrands as a sum of partial fractions and evaluate the integrals.

9. $\int \frac{dx}{1 - x^2}$

10. $\int \frac{dx}{x^2 + 2x}$

11. $\int \frac{x + 4}{x^2 + 5x - 6} dx$

12. $\int \frac{2x + 1}{x^2 - 7x + 12} dx$

13. $\int_4^8 \frac{y dy}{y^2 - 2y - 3}$

14. $\int_{1/2}^1 \frac{y + 4}{y^2 + y} dy$

15. $\int \frac{dt}{t^3 + t^2 - 2t}$

16. $\int \frac{x + 3}{2x^3 - 8x} dx$

Repeated Linear Factors

In Exercises 17–20, express the integrands as a sum of partial fractions and evaluate the integrals.

17. $\int_0^1 \frac{x^3 dx}{x^2 + 2x + 1}$

18. $\int_{-1}^0 \frac{x^3 dx}{x^2 - 2x + 1}$

19. $\int \frac{dx}{(x^2 - 1)^2}$

20. $\int \frac{x^2 dx}{(x - 1)(x^2 + 2x + 1)}$

Irreducible Quadratic Factors

In Exercises 21–28, express the integrands as a sum of partial fractions and evaluate the integrals.

21. $\int_0^1 \frac{dx}{(x + 1)(x^2 + 1)}$

22. $\int_1^{\sqrt{3}} \frac{3t^2 + t + 4}{t^3 + t} dt$

23. $\int \frac{y^2 + 2y + 1}{(y^2 + 1)^2} dy$

24. $\int \frac{8x^2 + 8x + 2}{(4x^2 + 1)^2} dx$

25. $\int \frac{2s + 2}{(s^2 + 1)(s - 1)^3} ds$

26. $\int \frac{s^4 + 81}{s(s^2 + 9)^2} ds$

27. $\int \frac{2\theta^3 + 5\theta^2 + 8\theta + 4}{(\theta^2 + 2\theta + 2)^2} d\theta$

28. $\int \frac{\theta^4 - 4\theta^3 + 2\theta^2 - 3\theta + 1}{(\theta^2 + 1)^3} d\theta$

Improper Fractions

In Exercises 29–34, perform long division on the integrand, write the proper fraction as a sum of partial fractions, and then evaluate the integral.

29. $\int \frac{2x^3 - 2x^2 + 1}{x^2 - x} dx$

30. $\int \frac{x^4}{x^2 - 1} dx$

31. $\int \frac{9x^3 - 3x + 1}{x^3 - x^2} dx$

32. $\int \frac{16x^3}{4x^2 - 4x + 1} dx$

33. $\int \frac{y^4 + y^2 - 1}{y^3 + y} dy$

34. $\int \frac{2y^4}{y^3 - y^2 + y - 1} dy$