Then

$$A^T = \begin{bmatrix} 4 & 0 \\ -2 & 5 \\ 3 & -2 \end{bmatrix}, \qquad B^T = \begin{bmatrix} 6 & 3 & 0 \\ 2 & -1 & 4 \\ -4 & 2 & 3 \end{bmatrix},$$

$$C^T = \begin{bmatrix} 5 & -3 & 2 \\ 4 & 2 & -3 \end{bmatrix}, \quad D^T = \begin{bmatrix} 3 \\ -5 \\ 1 \end{bmatrix}, \quad \text{and} \quad E^T = \begin{bmatrix} 2 & -1 & 3 \end{bmatrix}.$$

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Key Terms

Matrix
Rows
Columns
Size of a matrix
Square matrix
Main diagonal
Element or entry of a matrix
Equal matrices
n-vector (or vector)
R^n, C^n
0, zero vector
Google
Matrix addition
Scalar multiple
Difference of matrices
Summation notation
Index of summation
Linear combination
Coefficients
Transpose

1.2 Exercises

1. Let

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 6 & -5 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 4 \\ -3 \\ 5 \end{bmatrix},$$

and

$$C = \begin{bmatrix} 7 & 3 & 2 \\ -4 & 3 & 5 \\ 6 & 1 & -1 \end{bmatrix}.$$

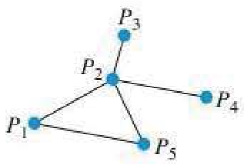
(a) What is a_{12}, a_{22}, a_{23}?

(b) What is b_{11}, b_{31}?

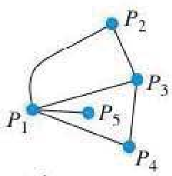
(c) What is c_{13}, c_{31}, c_{33}?

2. Determine the incidence matrix associated with each of the following graphs:

(a)



(b)



3. For each of the following incidence matrices, construct a graph. Label the vertices $P_1, P_2, \ldots, P_5$.

(a) $$A = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

(b) $$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

4. If

$$\begin{bmatrix} a+b & c+d \\ c-d & a-b \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 10 & 2 \end{bmatrix},$$

find a, b, c, and d.

5. If

$$\begin{bmatrix} a+2b & 2a-b \\ 2c+d & c-2d \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 4 & -3 \end{bmatrix},$$

find a, b, c, and d.

In Exercises 6 through 9, let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix},$$

$$C = \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix}, \quad D = \begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix},$$

$$E = \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}, \quad F = \begin{bmatrix} -4 & 5 \\ 2 & 3 \end{bmatrix},$$

$$\textit{and}\quad O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

6. If possible, compute the indicated linear combination:

(a) $C + E$ and $E + C$ **(b)** $A + B$

(c) $D - F$ **(d)** $-3C - 5O$

(e) $2C - 3E$ **(f)** $2B + F$

7. If possible, compute the indicated linear combination:

(a) $3D + 2F$ **(b)** $3(2A)$ and $6A$

(c) $3A + 2A$ and $5A$

(d) $2(D + F)$ and $2D + 2F$

(e) $(2 + 3)D$ and $2D + 3D$

(f) $3(B + D)$

8. If possible, compute the following:

(a) A^T and $(A^T)^T$

(b) $(C + E)^T$ and $C^T + E^T$

(c) $(2D + 3F)^T$ **(d)** $D - D^T$

(e) $2A^T + B$ **(f)** $(3D - 2F)^T$

9. If possible, compute the following:

(a) $(2A)^T$ **(b)** $(A - B)^T$

(c) $(3B^T - 2A)^T$ **(d)** $(3A^T - 5B^T)^T$

(e) $(-A)^T$ and $-(A^T)$ **(f)** $(C + E + F^T)^T$

10. Is the matrix $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$ a linear combination of the matrices $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$? Justify your answer.

11. Is the matrix $\begin{bmatrix} 4 & 1 \\ 0 & -3 \end{bmatrix}$ a linear combination of the matrices $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$? Justify your answer.

12. Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 6 & -2 & 3 \\ 5 & 2 & 4 \end{bmatrix} \quad \text{and} \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

If λ is a real number, compute $\lambda I_3 - A$.

13. If A is an $n \times n$ matrix, what are the entries on the main diagonal of $A - A^T$? Justify your answer.

14. Explain why every incidence matrix A associated with a graph is the same as A^T.

15. Let the $n \times n$ matrix A be equal to A^T. Briefly describe the pattern of the entries in A.

16. If $\mathbf{x}$ is an n-vector, show that $\mathbf{x} + \mathbf{0} = \mathbf{x}$.

17. Show that the summation notation satisfies the following properties:

(a) $\displaystyle\sum_{i=1}^{n} (r_i + s_i)a_i = \sum_{i=1}^{n} r_i a_i + \sum_{i=1}^{n} s_i a_i$

(b) $\displaystyle\sum_{i=1}^{n} c(r_i a_i) = c\left(\sum_{i=1}^{n} r_i a_i\right)$

18. Show that $\displaystyle\sum_{i=1}^{n}\left(\sum_{j=1}^{m} a_{ij}\right) = \sum_{j=1}^{m}\left(\sum_{i=1}^{n} a_{ij}\right)$.

19. Identify the following expressions as true or false. If true, prove the result; if false, give a counterexample.

(a) $\displaystyle\sum_{i=1}^{n} (a_i + 1) = \left(\sum_{i=1}^{n} a_i\right) + n$

(b) $\displaystyle\sum_{i=1}^{n}\left(\sum_{j=1}^{m} 1\right) = mn$

(c) $\displaystyle\sum_{j=1}^{m}\left(\sum_{i=1}^{n} a_i b_j\right) = \left[\sum_{i=1}^{n} a_i\right]\left[\sum_{j=1}^{m} b_j\right]$

20. A large steel manufacturer, who has 2000 employees, lists each employee's salary as a component of a vector $\mathbf{u}$ in R^{2000}. If an 8% across-the-board salary increase has been approved, find an expression involving $\mathbf{u}$ that gives all the new salaries.

21. A brokerage firm records the high and low values of the price of IBM stock each day. The information for a given week is presented in two vectors, $\mathbf{t}$ and $\mathbf{b}$, in R^5, showing the high and low values, respectively. What expression gives the average daily values of the price of IBM stock for the entire 5-day week?

22. For the software you are using, determine the commands to enter a matrix, add matrices, multiply a scalar times a matrix, and obtain the transpose of a matrix for matrices with numerical entries. Practice the commands, using the linear combinations in Example 13.

23. Determine whether the software you are using includes a computer algebra system (CAS), and if it does, do the following:

(a) Find the command for entering a symbolic matrix. (This command may be different than that for entering a numeric matrix.)

(b) Enter several symbolic matrices like

$$A = \begin{bmatrix} r & s & t \\ u & v & w \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}.$$

Compute expressions like $A + B$, $2A$, $3A + B$, $A - 2B$, $A^T + B^T$, etc. (In some systems you must