

## 5.3 THE DEFINITE INTEGRAL

1.  $\int_0^2 x^2 dx$

2.  $\int_{-1}^0 2x^3 dx$

3.  $\int_{-7}^5 (x^2 - 3x) dx$

4.  $\int_1^4 \frac{1}{x} dx$

5.  $\int_2^3 \frac{1}{1-x} dx$

6.  $\int_0^1 \sqrt{4-x^2} dx$

7.  $\int_{-\pi/4}^0 (\sec x) dx$

8.  $\int_0^{\pi/4} (\tan x) dx$

9. (a)  $\int_2^2 g(x) dx = 0$

(b)  $\int_5^1 g(x) dx = -\int_1^5 g(x) dx = -8$

(c)  $\int_1^2 3f(x) dx = 3 \int_1^2 f(x) dx = 3(-4) = -12$

(d)  $\int_2^5 f(x) dx = \int_1^5 f(x) dx - \int_1^2 f(x) dx = 6 - (-4) = 10$

(e)  $\int_1^5 [f(x) - g(x)] dx = \int_1^5 f(x) dx - \int_1^5 g(x) dx = 6 - 8 = -2$

(f)  $\int_1^5 [4f(x) - g(x)] dx = 4 \int_1^5 f(x) dx - \int_1^5 g(x) dx = 4(6) - 8 = 16$

10. (a)  $\int_1^9 -2f(x) dx = -2 \int_1^9 f(x) dx = -2(-1) = 2$

(b)  $\int_7^9 [f(x) + h(x)] dx = \int_7^9 f(x) dx + \int_7^9 h(x) dx = 5 + 4 = 9$

(c)  $\int_7^9 [2f(x) - 3h(x)] dx = 2 \int_7^9 f(x) dx - 3 \int_7^9 h(x) dx = 2(5) - 3(4) = -2$

(d)  $\int_9^1 f(x) dx = -\int_1^9 f(x) dx = -(-1) = 1$

(e)  $\int_1^7 f(x) dx = \int_1^9 f(x) dx - \int_7^9 f(x) dx = -1 - 5 = -6$

(f)  $\int_9^7 [h(x) - f(x)] dx = \int_7^9 [f(x) - h(x)] dx = \int_7^9 f(x) dx - \int_7^9 h(x) dx = 5 - 4 = 1$

11. (a)  $\int_1^2 f(u) du = \int_1^2 f(x) dx = 5$

(b)  $\int_1^2 \sqrt{3} f(z) dz = \sqrt{3} \int_1^2 f(z) dz = 5\sqrt{3}$

(c)  $\int_2^1 f(t) dt = -\int_1^2 f(t) dt = -5$

(d)  $\int_1^2 [-f(x)] dx = -\int_1^2 f(x) dx = -5$

12. (a)  $\int_0^{-3} g(t) dt = -\int_{-3}^0 g(t) dt = -\sqrt{2}$

(b)  $\int_{-3}^0 g(u) du = \int_{-3}^0 g(t) dt = \sqrt{2}$

(c)  $\int_{-3}^0 [-g(x)] dx = -\int_{-3}^0 g(x) dx = -\sqrt{2}$

(d)  $\int_{-3}^0 \frac{g(r)}{\sqrt{2}} dr = \frac{1}{\sqrt{2}} \int_{-3}^0 g(t) dt = \left(\frac{1}{\sqrt{2}}\right) (\sqrt{2}) = 1$

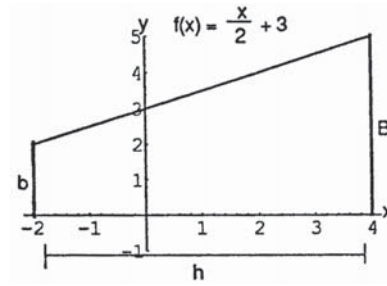
13. (a)  $\int_3^4 f(z) dz = \int_0^4 f(z) dz - \int_0^3 f(z) dz = 7 - 3 = 4$

(b)  $\int_4^3 f(t) dt = -\int_3^4 f(t) dt = -4$

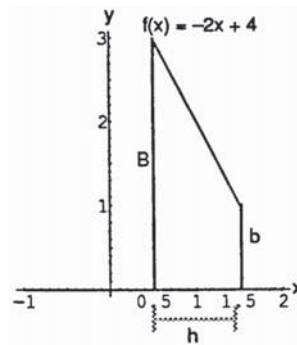
14. (a)  $\int_1^3 h(r) dr = \int_{-1}^3 h(r) dr - \int_{-1}^1 h(r) dr = 6 - 0 = 6$

(b)  $-\int_3^1 h(u) du = -\left(-\int_1^3 h(u) du\right) = \int_1^3 h(u) du = 6$

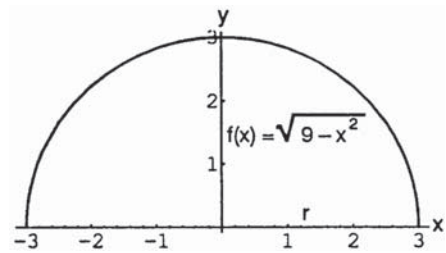
15. The area of the trapezoid is  $A = \frac{1}{2}(B + b)h$   
 $= \frac{1}{2}(5 + 2)(6) = 21 \Rightarrow \int_{-2}^4 \left(\frac{x}{2} + 3\right) dx$   
 $= 21$  square units



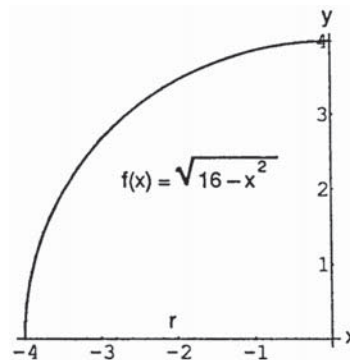
16. The area of the trapezoid is  $A = \frac{1}{2}(B + b)h$   
 $= \frac{1}{2}(3 + 1)(1) = 2 \Rightarrow \int_{1/2}^{3/2} (-2x + 4) dx$   
 $= 2$  square units



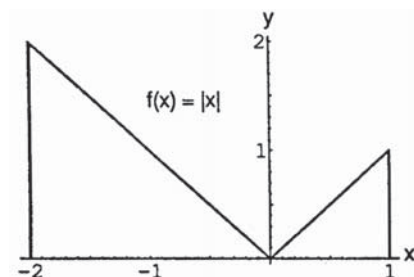
17. The area of the semicircle is  $A = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(3)^2$   
 $= \frac{9}{2}\pi \Rightarrow \int_{-3}^3 \sqrt{9 - x^2} dx = \frac{9}{2}\pi$  square units



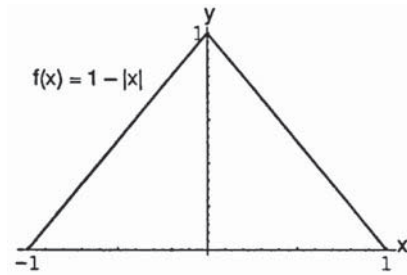
18. The graph of the quarter circle is  $A = \frac{1}{4}\pi r^2 = \frac{1}{4}\pi(4)^2$   
 $= 4\pi \Rightarrow \int_{-4}^0 \sqrt{16 - x^2} dx = 4\pi$  square units



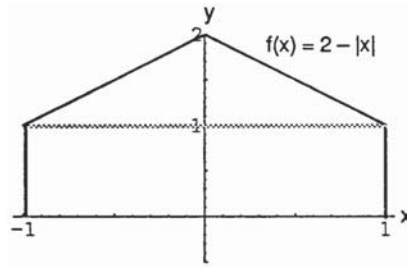
19. The area of the triangle on the left is  $A = \frac{1}{2}bh = \frac{1}{2}(2)(2) = 2$ . The area of the triangle on the right is  $A = \frac{1}{2}bh = \frac{1}{2}(1)(1) = \frac{1}{2}$ . Then, the total area is 2.5  
 $\Rightarrow \int_{-2}^1 |x| dx = 2.5$  square units



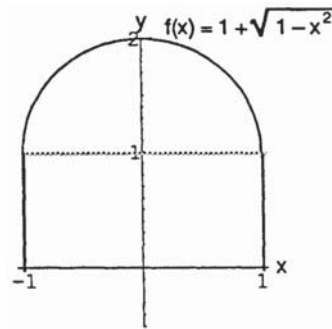
20. The area of the triangle is  $A = \frac{1}{2}bh = \frac{1}{2}(2)(1) = 1$   
 $\Rightarrow \int_{-1}^1 (1 - |x|) dx = 1$  square unit



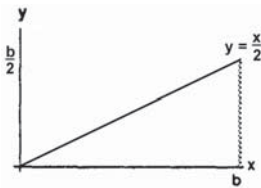
21. The area of the triangular peak is  $A = \frac{1}{2}bh = \frac{1}{2}(2)(1) = 1$ .  
 The area of the rectangular base is  $S = \ell w = (2)(1) = 2$ .  
 Then the total area is  $3 \Rightarrow \int_{-1}^1 (2 - |x|) dx = 3$  square units



22.  $y = 1 + \sqrt{1 - x^2} \Rightarrow y - 1 = \sqrt{1 - x^2}$   
 $\Rightarrow (y - 1)^2 = 1 - x^2 \Rightarrow x^2 + (y - 1)^2 = 1$ , a circle with center  $(0, 1)$  and radius of 1  $\Rightarrow y = 1 + \sqrt{1 - x^2}$  is the upper semicircle. The area of this semicircle is  $A = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(1)^2 = \frac{\pi}{2}$ . The area of the rectangular base is  $A = \ell w = (2)(1) = 2$ . Then the total area is  $2 + \frac{\pi}{2}$   
 $\Rightarrow \int_{-1}^1 (1 + \sqrt{1 - x^2}) dx = 2 + \frac{\pi}{2}$  square units



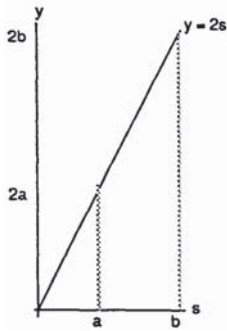
23.  $\int_0^b \frac{x}{2} dx = \frac{1}{2}(b)(\frac{b}{2}) = \frac{b^2}{4}$



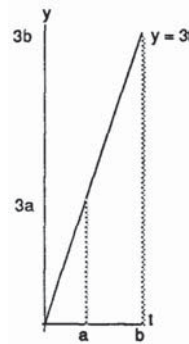
24.  $\int_0^b 4x dx = \frac{1}{2}b(4b) = 2b^2$



$$25. \int_a^b 2s \, ds = \frac{1}{2} b(2b) - \frac{1}{2} a(2a) = b^2 - a^2$$



$$26. \int_a^b 3t \, dt = \frac{1}{2} b(3b) - \frac{1}{2} a(3a) = \frac{3}{2} (b^2 - a^2)$$



$$27. \int_1^{\sqrt{2}} x \, dx = \frac{(\sqrt{2})^2}{2} - \frac{(1)^2}{2} = \frac{1}{2}$$

$$28. \int_{0.5}^{2.5} x \, dx = \frac{(2.5)^2}{2} - \frac{(0.5)^2}{2} = 3$$

$$29. \int_{\pi}^{2\pi} \theta \, d\theta = \frac{(2\pi)^2}{2} - \frac{\pi^2}{2} = \frac{3\pi^2}{2}$$

$$30. \int_{\sqrt{2}}^{5\sqrt{2}} r \, dr = \frac{(5\sqrt{2})^2}{2} - \frac{(\sqrt{2})^2}{2} = 24$$

$$31. \int_0^{\sqrt[3]{7}} x^2 \, dx = \frac{(\sqrt[3]{7})^3}{3} = \frac{7}{3}$$

$$32. \int_0^{0.3} s^2 \, ds = \frac{(0.3)^3}{3} = 0.009$$

$$33. \int_0^{1/2} t^2 \, dt = \frac{(\frac{1}{2})^3}{3} = \frac{1}{24}$$

$$34. \int_0^{\pi/2} \theta^2 \, d\theta = \frac{(\frac{\pi}{2})^3}{3} = \frac{\pi^3}{24}$$

$$35. \int_a^{2a} x \, dx = \frac{(2a)^2}{2} - \frac{a^2}{2} = \frac{3a^2}{2}$$

$$36. \int_a^{\sqrt{3}a} x \, dx = \frac{(\sqrt{3}a)^2}{2} - \frac{a^2}{2} = a^2$$

$$37. \int_0^{\sqrt[3]{b}} x^2 \, dx = \frac{(\sqrt[3]{b})^3}{3} = \frac{b}{3}$$

$$38. \int_0^{3b} x^2 \, dx = \frac{(3b)^3}{3} = 9b^3$$

$$39. \int_3^1 7 \, dx = 7(1 - 3) = -14$$

$$40. \int_0^{-2} \sqrt{2} \, dx = \sqrt{2}(-2 - 0) = -2\sqrt{2}$$

$$41. \int_0^2 5x \, dx = 5 \int_0^2 x \, dx = 5 \left[ \frac{2^2}{2} - \frac{0^2}{2} \right] = 10$$

$$42. \int_3^5 \frac{x}{8} \, dx = \frac{1}{8} \int_3^5 x \, dx = \frac{1}{8} \left[ \frac{5^2}{2} - \frac{3^2}{2} \right] = \frac{16}{16} = 1$$

$$43. \int_0^2 (2t - 3) \, dt = 2 \int_0^2 t \, dt - \int_0^2 3 \, dt = 2 \left[ \frac{2^2}{2} - \frac{0^2}{2} \right] - 3(2 - 0) = 4 - 6 = -2$$

$$44. \int_0^{\sqrt{2}} (t - \sqrt{2}) \, dt = \int_0^{\sqrt{2}} t \, dt - \int_0^{\sqrt{2}} \sqrt{2} \, dt = \left[ \frac{(\sqrt{2})^2}{2} - \frac{0^2}{2} \right] - \sqrt{2} [\sqrt{2} - 0] = 1 - 2 = -1$$

$$45. \int_2^1 \left(1 + \frac{z}{2}\right) \, dz = \int_2^1 1 \, dz + \int_2^1 \frac{z}{2} \, dz = \int_2^1 1 \, dz - \frac{1}{2} \int_1^2 z \, dz = 1[1 - 2] - \frac{1}{2} \left[ \frac{2^2}{2} - \frac{1^2}{2} \right] = -1 - \frac{1}{2} \left( \frac{3}{2} \right) = -\frac{7}{4}$$

$$46. \int_3^0 (2z - 3) \, dz = \int_3^0 2z \, dz - \int_3^0 3 \, dz = -2 \int_0^3 z \, dz - \int_3^0 3 \, dz = -2 \left[ \frac{3^2}{2} - \frac{0^2}{2} \right] - 3[0 - 3] = -9 + 9 = 0$$

$$47. \int_1^2 3u^2 \, du = 3 \int_1^2 u^2 \, du = 3 \left[ \int_0^2 u^2 \, du - \int_0^1 u^2 \, du \right] = 3 \left( \left[ \frac{2^3}{3} - \frac{0^3}{3} \right] - \left[ \frac{1^3}{3} - \frac{0^3}{3} \right] \right) = 3 \left[ \frac{2^3}{3} - \frac{1^3}{3} \right] = 3 \left( \frac{7}{3} \right) = 7$$

$$48. \int_{1/2}^1 24u^2 \, du = 24 \int_{1/2}^1 u^2 \, du = 24 \left[ \int_0^1 u^2 \, du - \int_0^{1/2} u^2 \, du \right] = 24 \left[ \frac{1^3}{3} - \frac{(\frac{1}{2})^3}{3} \right] = 24 \left[ \frac{(\frac{7}{8})}{3} \right] = 7$$

$$49. \int_0^2 (3x^2 + x - 5) \, dx = 3 \int_0^2 x^2 \, dx + \int_0^2 x \, dx - \int_0^2 5 \, dx = 3 \left[ \frac{2^3}{3} - \frac{0^3}{3} \right] + \left[ \frac{2^2}{2} - \frac{0^2}{2} \right] - 5[2 - 0] = (8 + 2) - 10 = 0$$

$$50. \int_1^0 (3x^2 + x - 5) \, dx = - \int_0^1 (3x^2 + x - 5) \, dx = - \left[ 3 \int_0^1 x^2 \, dx + \int_0^1 x \, dx - \int_0^1 5 \, dx \right]$$

$$= - \left[ 3 \left( \frac{1^3}{3} - \frac{0^3}{3} \right) + \left( \frac{1^2}{2} - \frac{0^2}{2} \right) - 5(1 - 0) \right] = - \left( \frac{3}{2} - 5 \right) = \frac{7}{2}$$

51. Let  $\Delta x = \frac{b-0}{n} = \frac{b}{n}$  and let  $x_0 = 0, x_1 = \Delta x,$   
 $x_2 = 2\Delta x, \dots, x_{n-1} = (n-1)\Delta x, x_n = n\Delta x = b.$   
 Let the  $c_k$ 's be the right end-points of the subintervals  
 $\Rightarrow c_1 = x_1, c_2 = x_2,$  and so on. The rectangles  
 defined have areas:

$$f(c_1) \Delta x = f(\Delta x) \Delta x = 3(\Delta x)^2 \Delta x = 3(\Delta x)^3$$

$$f(c_2) \Delta x = f(2\Delta x) \Delta x = 3(2\Delta x)^2 \Delta x = 3(2)^2(\Delta x)^3$$

$$f(c_3) \Delta x = f(3\Delta x) \Delta x = 3(3\Delta x)^2 \Delta x = 3(3)^2(\Delta x)^3$$

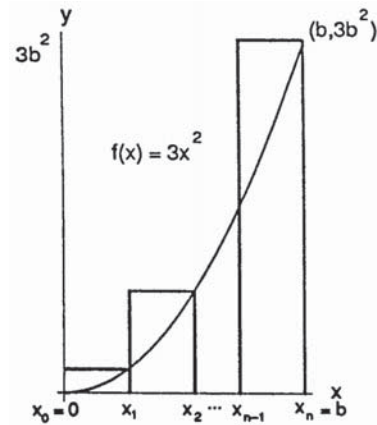
$$\vdots$$

$$f(c_n) \Delta x = f(n\Delta x) \Delta x = 3(n\Delta x)^2 \Delta x = 3(n)^2(\Delta x)^3$$

$$\text{Then } S_n = \sum_{k=1}^n f(c_k) \Delta x = \sum_{k=1}^n 3k^2(\Delta x)^3$$

$$= 3(\Delta x)^3 \sum_{k=1}^n k^2 = 3 \left( \frac{b^3}{n^3} \right) \left( \frac{n(n+1)(2n+1)}{6} \right)$$

$$= \frac{b^3}{2} \left( 2 + \frac{3}{n} + \frac{1}{n^2} \right) \Rightarrow \int_0^b 3x^2 \, dx = \lim_{n \rightarrow \infty} \frac{b^3}{2} \left( 2 + \frac{3}{n} + \frac{1}{n^2} \right) = b^3.$$



52. Let  $\Delta x = \frac{b-0}{n} = \frac{b}{n}$  and let  $x_0 = 0, x_1 = \Delta x,$   
 $x_2 = 2\Delta x, \dots, x_{n-1} = (n-1)\Delta x, x_n = n\Delta x = b.$   
 Let the  $c_k$ 's be the right end-points of the subintervals  
 $\Rightarrow c_1 = x_1, c_2 = x_2,$  and so on. The rectangles  
 defined have areas:

$$f(c_1) \Delta x = f(\Delta x) \Delta x = \pi(\Delta x)^2 \Delta x = \pi(\Delta x)^3$$

$$f(c_2) \Delta x = f(2\Delta x) \Delta x = \pi(2\Delta x)^2 \Delta x = \pi(2)^2(\Delta x)^3$$

$$f(c_3) \Delta x = f(3\Delta x) \Delta x = \pi(3\Delta x)^2 \Delta x = \pi(3)^2(\Delta x)^3$$

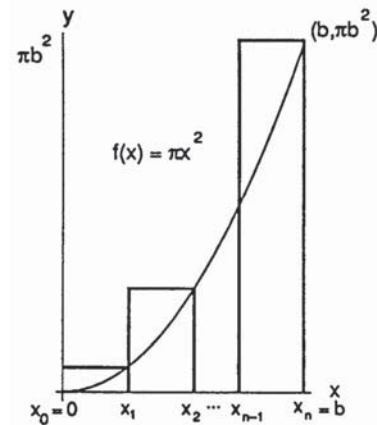
$$\vdots$$

$$f(c_n) \Delta x = f(n\Delta x) \Delta x = \pi(n\Delta x)^2 \Delta x = \pi(n)^2(\Delta x)^3$$

$$\text{Then } S_n = \sum_{k=1}^n f(c_k) \Delta x = \sum_{k=1}^n \pi k^2(\Delta x)^3$$

$$= \pi(\Delta x)^3 \sum_{k=1}^n k^2 = \pi \left( \frac{b^3}{n^3} \right) \left( \frac{n(n+1)(2n+1)}{6} \right)$$

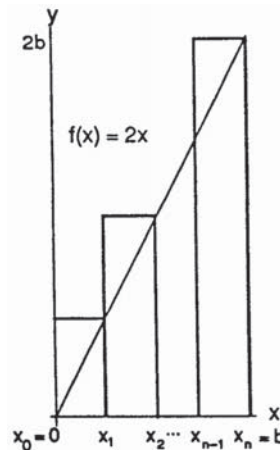
$$= \frac{\pi b^3}{6} \left( 2 + \frac{3}{n} + \frac{1}{n^2} \right) \Rightarrow \int_0^b \pi x^2 \, dx = \lim_{n \rightarrow \infty} \frac{\pi b^3}{6} \left( 2 + \frac{3}{n} + \frac{1}{n^2} \right) = \frac{\pi b^3}{3}.$$



53. Let  $\Delta x = \frac{b-0}{n} = \frac{b}{n}$  and let  $x_0 = 0, x_1 = \Delta x,$   
 $x_2 = 2\Delta x, \dots, x_{n-1} = (n-1)\Delta x, x_n = n\Delta x = b.$   
 Let the  $c_k$ 's be the right end-points of the subintervals  
 $\Rightarrow c_1 = x_1, c_2 = x_2,$  and so on. The rectangles  
 defined have areas:

$$\begin{aligned} f(c_1) \Delta x &= f(\Delta x) \Delta x = 2(\Delta x)(\Delta x) = 2(\Delta x)^2 \\ f(c_2) \Delta x &= f(2\Delta x) \Delta x = 2(2\Delta x)(\Delta x) = 2(2)(\Delta x)^2 \\ f(c_3) \Delta x &= f(3\Delta x) \Delta x = 2(3\Delta x)(\Delta x) = 2(3)(\Delta x)^2 \\ &\vdots \\ f(c_n) \Delta x &= f(n\Delta x) \Delta x = 2(n\Delta x)(\Delta x) = 2(n)(\Delta x)^2 \end{aligned}$$

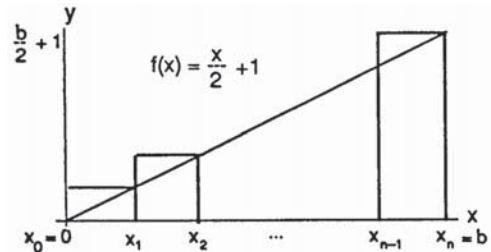
$$\begin{aligned} \text{Then } S_n &= \sum_{k=1}^n f(c_k) \Delta x = \sum_{k=1}^n 2k(\Delta x)^2 \\ &= 2(\Delta x)^2 \sum_{k=1}^n k = 2 \left( \frac{b^2}{n^2} \right) \left( \frac{n(n+1)}{2} \right) \\ &= b^2 \left( 1 + \frac{1}{n} \right) \Rightarrow \int_0^b 2x \, dx = \lim_{n \rightarrow \infty} b^2 \left( 1 + \frac{1}{n} \right) = b^2. \end{aligned}$$



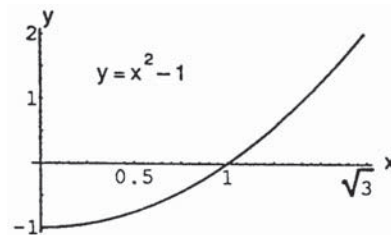
54. Let  $\Delta x = \frac{b-0}{n} = \frac{b}{n}$  and let  $x_0 = 0, x_1 = \Delta x,$   
 $x_2 = 2\Delta x, \dots, x_{n-1} = (n-1)\Delta x, x_n = n\Delta x = b.$   
 Let the  $c_k$ 's be the right end-points of the subintervals  
 $\Rightarrow c_1 = x_1, c_2 = x_2,$  and so on. The rectangles  
 defined have areas:

$$\begin{aligned} f(c_1) \Delta x &= f(\Delta x) \Delta x = \left( \frac{\Delta x}{2} + 1 \right) (\Delta x) = \frac{1}{2} (\Delta x)^2 + \Delta x \\ f(c_2) \Delta x &= f(2\Delta x) \Delta x = \left( \frac{2\Delta x}{2} + 1 \right) (\Delta x) = \frac{1}{2} (2)(\Delta x)^2 + \Delta x \\ f(c_3) \Delta x &= f(3\Delta x) \Delta x = \left( \frac{3\Delta x}{2} + 1 \right) (\Delta x) = \frac{1}{2} (3)(\Delta x)^2 + \Delta x \\ &\vdots \\ f(c_n) \Delta x &= f(n\Delta x) \Delta x = \left( \frac{n\Delta x}{2} + 1 \right) (\Delta x) = \frac{1}{2} (n)(\Delta x)^2 + \Delta x \end{aligned}$$

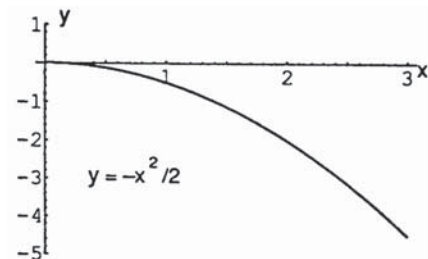
$$\begin{aligned} \text{Then } S_n &= \sum_{k=1}^n f(c_k) \Delta x = \sum_{k=1}^n \left( \frac{1}{2} k(\Delta x)^2 + \Delta x \right) = \frac{1}{2} (\Delta x)^2 \sum_{k=1}^n k + \Delta x \sum_{k=1}^n 1 = \frac{1}{2} \left( \frac{b^2}{n^2} \right) \left( \frac{n(n+1)}{2} \right) + \left( \frac{b}{n} \right) (n) \\ &= \frac{1}{4} b^2 \left( 1 + \frac{1}{n} \right) + b \Rightarrow \int_0^b \left( \frac{x}{2} + 1 \right) dx = \lim_{n \rightarrow \infty} \left( \frac{1}{4} b^2 \left( 1 + \frac{1}{n} \right) + b \right) = \frac{1}{4} b^2 + b. \end{aligned}$$



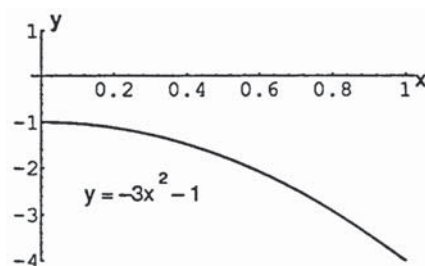
55. 
$$\begin{aligned} \text{av}(f) &= \left( \frac{1}{\sqrt{3}-0} \right) \int_0^{\sqrt{3}} (x^2 - 1) \, dx \\ &= \frac{1}{\sqrt{3}} \int_0^{\sqrt{3}} x^2 \, dx - \frac{1}{\sqrt{3}} \int_0^{\sqrt{3}} 1 \, dx \\ &= \frac{1}{\sqrt{3}} \left( \frac{(\sqrt{3})^3}{3} \right) - \frac{1}{\sqrt{3}} (\sqrt{3} - 0) = 1 - 1 = 0. \end{aligned}$$



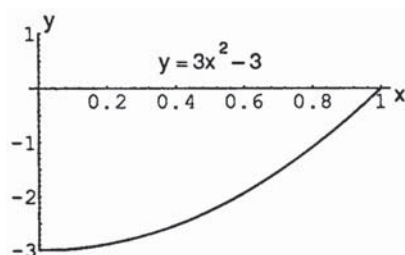
56. 
$$\begin{aligned} \text{av}(f) &= \left( \frac{1}{3-0} \right) \int_0^3 \left( -\frac{x^2}{2} \right) dx = \frac{1}{3} \left( -\frac{1}{2} \right) \int_0^3 x^2 \, dx \\ &= -\frac{1}{6} \left( \frac{3^3}{3} \right) = -\frac{3}{2}; -\frac{x^2}{2} = -\frac{3}{2}. \end{aligned}$$



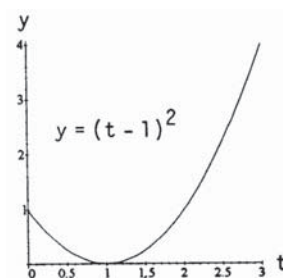
$$\begin{aligned}
 57. \text{av}(f) &= \left(\frac{1}{1-0}\right) \int_0^1 (-3x^2 - 1) \, dx = \\
 &= -3 \int_0^1 x^2 \, dx - \int_0^1 1 \, dx = -3 \left(\frac{1^3}{3}\right) - (1 - 0) \\
 &= -2.
 \end{aligned}$$



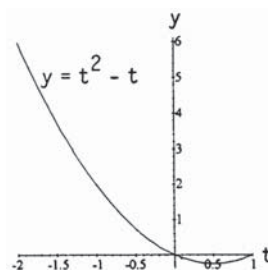
$$\begin{aligned}
 58. \text{av}(f) &= \left(\frac{1}{1-0}\right) \int_0^1 (3x^2 - 3) \, dx = \\
 &= 3 \int_0^1 x^2 \, dx - \int_0^1 3 \, dx = 3 \left(\frac{1^3}{3}\right) - 3(1 - 0) \\
 &= -2.
 \end{aligned}$$



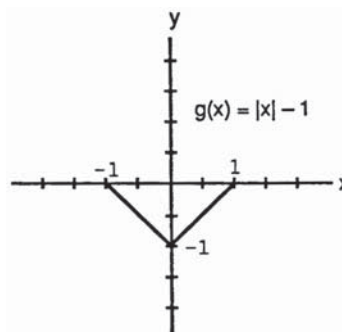
$$\begin{aligned}
 59. \text{av}(f) &= \left(\frac{1}{3-0}\right) \int_0^3 (t-1)^2 \, dt \\
 &= \frac{1}{3} \int_0^3 t^2 \, dt - \frac{2}{3} \int_0^3 t \, dt + \frac{1}{3} \int_0^3 1 \, dt \\
 &= \frac{1}{3} \left(\frac{3^3}{3}\right) - \frac{2}{3} \left(\frac{3^2}{2} - \frac{0^2}{2}\right) + \frac{1}{3} (3 - 0) = 1.
 \end{aligned}$$



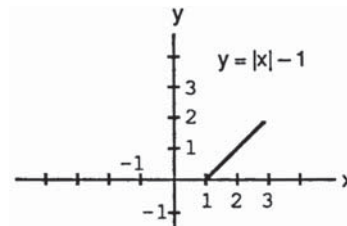
$$\begin{aligned}
 60. \text{av}(f) &= \left(\frac{1}{1-(-2)}\right) \int_{-2}^1 (t^2 - t) \, dt \\
 &= \frac{1}{3} \int_{-2}^1 t^2 \, dt - \frac{1}{3} \int_{-2}^1 t \, dt \\
 &= \frac{1}{3} \int_0^1 t^2 \, dt - \frac{1}{3} \int_0^{-2} t^2 \, dt - \frac{1}{3} \left(\frac{1^2}{2} - \frac{(-2)^2}{2}\right) \\
 &= \frac{1}{3} \left(\frac{1^3}{3}\right) - \frac{1}{3} \left(\frac{(-2)^3}{3}\right) + \frac{1}{2} = \frac{3}{2}.
 \end{aligned}$$



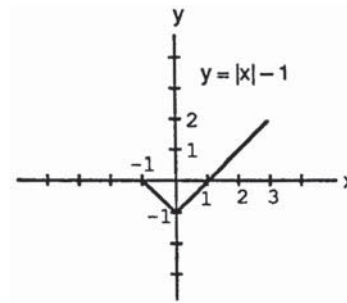
$$\begin{aligned}
 61. (a) \text{av}(g) &= \left(\frac{1}{1-(-1)}\right) \int_{-1}^1 (|x| - 1) \, dx \\
 &= \frac{1}{2} \int_{-1}^0 (-x - 1) \, dx + \frac{1}{2} \int_0^1 (x - 1) \, dx \\
 &= -\frac{1}{2} \int_{-1}^0 x \, dx - \frac{1}{2} \int_{-1}^0 1 \, dx + \frac{1}{2} \int_0^1 x \, dx - \frac{1}{2} \int_0^1 1 \, dx \\
 &= -\frac{1}{2} \left(\frac{0^2}{2} - \frac{(-1)^2}{2}\right) - \frac{1}{2} (0 - (-1)) + \frac{1}{2} \left(\frac{1^2}{2} - \frac{0^2}{2}\right) - \frac{1}{2} (1 - 0) \\
 &= -\frac{1}{2}.
 \end{aligned}$$



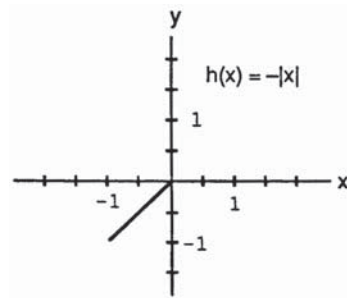
$$\begin{aligned}
 \text{(b) } \text{av}(g) &= \left(\frac{1}{3-1}\right) \int_1^3 (|x| - 1) \, dx = \frac{1}{2} \int_1^3 (x - 1) \, dx \\
 &= \frac{1}{2} \int_1^3 x \, dx - \frac{1}{2} \int_1^3 1 \, dx = \frac{1}{2} \left(\frac{3^2}{2} - \frac{1^2}{2}\right) - \frac{1}{2}(3 - 1) \\
 &= 1.
 \end{aligned}$$



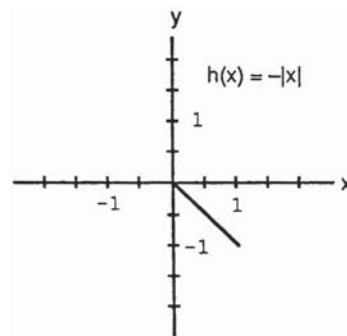
$$\begin{aligned}
 \text{(c) } \text{av}(g) &= \left(\frac{1}{3-(-1)}\right) \int_{-1}^3 (|x| - 1) \, dx \\
 &= \frac{1}{4} \int_{-1}^1 (|x| - 1) \, dx + \frac{1}{4} \int_1^3 (|x| - 1) \, dx \\
 &= \frac{1}{4}(-1 + 2) = \frac{1}{4} \text{ (see parts (a) and (b) above).}
 \end{aligned}$$



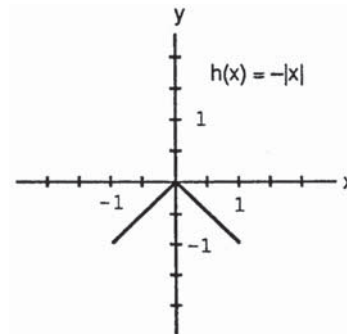
$$\begin{aligned}
 62. \text{ (a) } \text{av}(h) &= \left(\frac{1}{0-(-1)}\right) \int_{-1}^0 -|x| \, dx = \int_{-1}^0 -(-x) \, dx \\
 &= \int_{-1}^0 x \, dx = \frac{0^2}{2} - \frac{(-1)^2}{2} = -\frac{1}{2}.
 \end{aligned}$$



$$\begin{aligned}
 \text{(b) } \text{av}(h) &= \left(\frac{1}{1-0}\right) \int_0^1 -|x| \, dx = -\int_0^1 x \, dx \\
 &= -\left(\frac{1^2}{2} - \frac{0^2}{2}\right) = -\frac{1}{2}.
 \end{aligned}$$



$$\begin{aligned}
 \text{(c) } \text{av}(h) &= \left(\frac{1}{1-(-1)}\right) \int_{-1}^1 -|x| \, dx \\
 &= \frac{1}{2} \left( \int_{-1}^0 -|x| \, dx + \int_0^1 -|x| \, dx \right) \\
 &= \frac{1}{2} \left(-\frac{1}{2} + \left(-\frac{1}{2}\right)\right) = -\frac{1}{2} \text{ (see parts (a) and (b) above).}
 \end{aligned}$$



63. To find where  $x - x^2 \geq 0$ , let  $x - x^2 = 0 \Rightarrow x(1 - x) = 0 \Rightarrow x = 0$  or  $x = 1$ . If  $0 < x < 1$ , then  $0 < x - x^2 \Rightarrow a = 0$  and  $b = 1$  maximize the integral.