$\Rightarrow$ $s = 8t - \cos(\pi t) + 1 \Rightarrow s(1) = 8 - \cos\pi + 1 = 10$ m

61. All three integrations are correct. In each case, the derivative of the function on the right is the integrand on the left, and each formula has an arbitrary constant for generating the remaining antiderivatives. Moreover, $\sin^2 x + C_1 = 1 - \cos^2 x + C_1 \Rightarrow C_2 = 1 + C_1$; also $-\cos^2 x + C_2 = -\frac{\cos 2x}{2} - \frac{1}{2} + C_2 \Rightarrow C_3 = C_2 - \frac{1}{2} = C_1 + \frac{1}{2}$.

62. Both integrations are correct. In each case, the derivative of the function on the right is the integrand on the left, and each formula has an arbitrary constant for generating the remaining antiderivatives. Moreover,
$$\frac{\tan^2 x}{2} + C = \frac{\sec^2 x - 1}{2} + C = \frac{\sec^2 x}{2} + \underbrace{\left(C - \frac{1}{2}\right)}_{\text{a constant}}$$

63. (a) $\left(\frac{1}{\frac{1}{60} - 0}\right)\int_0^{1/60} V_{max}\sin 120\pi t\,dt = 60\left[-V_{max}\left(\frac{1}{120\pi}\right)\cos(120\pi t)\right]_0^{1/60} = -\frac{V_{max}}{2\pi}[\cos 2\pi - \cos 0]$
$= -\frac{V_{max}}{2\pi}[1 - 1] = 0$

(b) $V_{max} = \sqrt{2}\,V_{rms} = \sqrt{2}(240) \approx 339$ volts

(c) $\int_0^{1/60}(V_{max})^2\sin^2 120\pi t\,dt = (V_{max})^2\int_0^{1/60}\left(\frac{1 - \cos 240\pi t}{2}\right)dt = \frac{(V_{max})^2}{2}\int_0^{1/60}(1 - \cos 240\pi t)\,dt$
$= \frac{(V_{max})^2}{2}\left[t - \left(\frac{1}{240\pi}\right)\sin 240\pi t\right]_0^{1/60} = \frac{(V_{max})^2}{2}\left[\left(\frac{1}{60} - \left(\frac{1}{240\pi}\right)\sin(4\pi)\right) - \left(0 - \left(\frac{1}{240\pi}\right)\sin(0)\right)\right] = \frac{(V_{max})^2}{120}$

5.6 SUBSTITUTION AND AREA BETWEEN CURVES

1. (a) Let $u = y + 1 \Rightarrow du = dy$; $y = 0 \Rightarrow u = 1$, $y = 3 \Rightarrow u = 4$
$\int_0^3\sqrt{y+1}\,dy = \int_1^4 u^{1/2}\,du = \left[\frac{2}{3}u^{3/2}\right]_1^4 = \left(\frac{2}{3}\right)(4)^{3/2} - \left(\frac{2}{3}\right)(1)^{3/2} = \left(\frac{2}{3}\right)(8) - \left(\frac{2}{3}\right)(1) = \frac{14}{3}$

(b) Use the same substitution for u as in part (a); $y = -1 \Rightarrow u = 0$, $y = 0 \Rightarrow u = 1$
$\int_{-1}^0\sqrt{y+1}\,dy = \int_0^1 u^{1/2}\,du = \left[\frac{2}{3}u^{3/2}\right]_0^1 = \left(\frac{2}{3}\right)(1)^{3/2} - 0 = \frac{2}{3}$

2. (a) Let $u = 1 - r^2 \Rightarrow du = -2r\,dr \Rightarrow -\frac{1}{2}du = r\,dr$; $r = 0 \Rightarrow u = 1$, $r = 1 \Rightarrow u = 0$
$\int_0^1 r\sqrt{1 - r^2}\,dr = \int_1^0 -\frac{1}{2}\sqrt{u}\,du = \left[-\frac{1}{3}u^{3/2}\right]_1^0 = 0 - \left(-\frac{1}{3}\right)(1)^{3/2} = \frac{1}{3}$

(b) Use the same substitution for u as in part (a); $r = -1 \Rightarrow u = 0$, $r = 1 \Rightarrow u = 0$
$\int_{-1}^1 r\sqrt{1 - r^2}\,dr = \int_0^0 -\frac{1}{2}\sqrt{u}\,du = 0$

3. (a) Let $u = \tan x \Rightarrow du = \sec^2 x\,dx$; $x = 0 \Rightarrow u = 0$, $x = \frac{\pi}{4} \Rightarrow u = 1$
$\int_0^{\pi/4}\tan x\sec^2 x\,dx = \int_0^1 u\,du = \left[\frac{u^2}{2}\right]_0^1 = \frac{1^2}{2} - 0 = \frac{1}{2}$

(b) Use the same substitution as in part (a); $x = -\frac{\pi}{4} \Rightarrow u = -1$, $x = 0 \Rightarrow u = 0$
$\int_{-\pi/4}^0\tan x\sec^2 x\,dx = \int_{-1}^0 u\,du = \left[\frac{u^2}{2}\right]_{-1}^0 = 0 - \frac{1}{2} = -\frac{1}{2}$

4. (a) Let $u = \cos x \Rightarrow du = -\sin x\,dx \Rightarrow -du = \sin x\,dx$; $x = 0 \Rightarrow u = 1$, $x = \pi \Rightarrow u = -1$
$\int_0^{\pi}3\cos^2 x\sin x\,dx = \int_1^{-1} -3u^2\,du = \left[-u^3\right]_1^{-1} = -(-1)^3 - \left(-(1)^3\right) = 2$

(b) Use the same substitution as in part (a); $x = 2\pi \Rightarrow u = 1$, $x = 3\pi \Rightarrow u = -1$
$\int_{2\pi}^{3\pi}3\cos^2 x\sin x\,dx = \int_1^{-1} -3u^2\,du = 2$

5. (a) $u = 1 + t^4 \Rightarrow du = 4t^3\,dt \Rightarrow \frac{1}{4}\,du = t^3\,dt;\ t = 0 \Rightarrow u = 1,\ t = 1 \Rightarrow u = 2$

$\int_0^1 t^3\left(1+t^4\right)^3 dt = \int_1^2 \frac{1}{4}\,u^3\,du = \left[\frac{u^4}{16}\right]_1^2 = \frac{2^4}{16} - \frac{1^4}{16} = \frac{15}{16}$

(b) Use the same substitution as in part (a); $t = -1 \Rightarrow u = 2,\ t = 1 \Rightarrow u = 2$

$\int_{-1}^1 t^3\left(1+t^4\right)^3 dt = \int_2^2 \frac{1}{4}\,u^3\,du = 0$

6. (a) Let $u = t^2 + 1 \Rightarrow du = 2t\,dt \Rightarrow \frac{1}{2}\,du = t\,dt;\ t = 0 \Rightarrow u = 1,\ t = \sqrt{7} \Rightarrow u = 8$

$\int_0^{\sqrt{7}} t\left(t^2+1\right)^{1/3} dt = \int_1^8 \frac{1}{2}\,u^{1/3}\,du = \left[\left(\frac{1}{2}\right)\left(\frac{3}{4}\right)u^{4/3}\right]_1^8 = \left(\frac{3}{8}\right)(8)^{4/3} - \left(\frac{3}{8}\right)(1)^{4/3} = \frac{45}{8}$

(b) Use the same substitution as in part (a); $t = -\sqrt{7} \Rightarrow u = 8,\ t = 0 \Rightarrow u = 1$

$\int_{-\sqrt{7}}^0 t\left(t^2+1\right)^{1/3} dt = \int_8^1 \frac{1}{2}\,u^{1/3}\,du = -\int_1^8 \frac{1}{2}\,u^{1/3}\,du = -\frac{45}{8}$

7. (a) Let $u = 4 + r^2 \Rightarrow du = 2r\,dr \Rightarrow \frac{1}{2}\,du = r\,dr;\ r = -1 \Rightarrow u = 5,\ r = 1 \Rightarrow u = 5$

$\int_{-1}^1 \frac{5r}{\left(4+r^2\right)^2}\,dr = 5\int_5^5 \frac{1}{2}\,u^{-2}\,du = 0$

(b) Use the same substitution as in part (a); $r = 0 \Rightarrow u = 4,\ r = 1 \Rightarrow u = 5$

$\int_0^1 \frac{5r}{\left(4+r^2\right)^2}\,dr = 5\int_4^5 \frac{1}{2}\,u^{-2}\,du = 5\left[-\frac{1}{2}\,u^{-1}\right]_4^5 = 5\left(-\frac{1}{2}\,(5)^{-1}\right) - 5\left(-\frac{1}{2}\,(4)^{-1}\right) = \frac{1}{8}$

8. (a) Let $u = 1 + v^{3/2} \Rightarrow du = \frac{3}{2}\,v^{1/2}\,dv \Rightarrow \frac{20}{3}\,du = 10\sqrt{v}\,dv;\ v = 0 \Rightarrow u = 1,\ v = 1 \Rightarrow u = 2$

$\int_0^1 \frac{10\sqrt{v}}{\left(1+v^{3/2}\right)^2}\,dv = \int_1^2 \frac{1}{u^2}\left(\frac{20}{3}\,du\right) = \frac{20}{3}\int_1^2 u^{-2}\,du = -\frac{20}{3}\left[\frac{1}{u}\right]_1^2 = -\frac{20}{3}\left[\frac{1}{2} - \frac{1}{1}\right] = \frac{10}{3}$

(b) Use the same substitution as in part (a); $v = 1 \Rightarrow u = 2,\ v = 4 \Rightarrow u = 1 + 4^{3/2} = 9$

$\int_1^4 \frac{10\sqrt{v}}{\left(1+v^{3/2}\right)^2}\,dv = \int_2^9 \frac{1}{u^2}\left(\frac{20}{3}\,du\right) = -\frac{20}{3}\left[\frac{1}{u}\right]_2^9 = -\frac{20}{3}\left(\frac{1}{9} - \frac{1}{2}\right) = -\frac{20}{3}\left(-\frac{7}{18}\right) = \frac{70}{27}$

9. (a) Let $u = x^2 + 1 \Rightarrow du = 2x\,dx \Rightarrow 2\,du = 4x\,dx;\ x = 0 \Rightarrow u = 1,\ x = \sqrt{3} \Rightarrow u = 4$

$\int_0^{\sqrt{3}} \frac{4x}{\sqrt{x^2+1}}\,dx = \int_1^4 \frac{2}{\sqrt{u}}\,du = \int_1^4 2u^{-1/2}\,du = \left[4u^{1/2}\right]_1^4 = 4(4)^{1/2} - 4(1)^{1/2} = 4$

(b) Use the same substitution as in part (a); $x = -\sqrt{3} \Rightarrow u = 4,\ x = \sqrt{3} \Rightarrow u = 4$

$\int_{-\sqrt{3}}^{\sqrt{3}} \frac{4x}{\sqrt{x^2+1}}\,dx = \int_4^4 \frac{2}{\sqrt{u}}\,du = 0$

10. (a) Let $u = x^4 + 9 \Rightarrow du = 4x^3\,dx \Rightarrow \frac{1}{4}\,du = x^3\,dx;\ x = 0 \Rightarrow u = 9,\ x = 1 \Rightarrow u = 10$

$\int_0^1 \frac{x^3}{\sqrt{x^4+9}}\,dx = \int_9^{10} \frac{1}{4}\,u^{-1/2}\,du = \left[\frac{1}{4}\,(2)u^{1/2}\right]_9^{10} = \frac{1}{2}\,(10)^{1/2} - \frac{1}{2}\,(9)^{1/2} = \frac{\sqrt{10}-3}{2}$

(b) Use the same substitution as in part (a); $x = -1 \Rightarrow u = 10,\ x = 0 \Rightarrow u = 9$

$\int_{-1}^0 \frac{x^3}{\sqrt{x^4+9}}\,dx = \int_{10}^9 \frac{1}{4}\,u^{-1/2}\,du = -\int_9^{10} \frac{1}{4}\,u^{-1/2}\,du = \frac{3-\sqrt{10}}{2}$

11. (a) Let $u = 1 - \cos 3t \Rightarrow du = 3\sin 3t\,dt \Rightarrow \frac{1}{3}\,du = \sin 3t\,dt;\ t = 0 \Rightarrow u = 0,\ t = \frac{\pi}{6} \Rightarrow u = 1 - \cos\frac{\pi}{2} = 1$

$\int_0^{\pi/6} (1 - \cos 3t)\sin 3t\,dt = \int_0^1 \frac{1}{3}\,u\,du = \left[\frac{1}{3}\left(\frac{u^2}{2}\right)\right]_0^1 = \frac{1}{6}\,(1)^2 - \frac{1}{6}\,(0)^2 = \frac{1}{6}$

(b) Use the same substitution as in part (a); $t = \frac{\pi}{6} \Rightarrow u = 1,\ t = \frac{\pi}{3} \Rightarrow u = 1 - \cos\pi = 2$

$\int_{\pi/6}^{\pi/3} (1 - \cos 3t)\sin 3t\,dt = \int_1^2 \frac{1}{3}\,u\,du = \left[\frac{1}{3}\left(\frac{u^2}{2}\right)\right]_1^2 = \frac{1}{6}\,(2)^2 - \frac{1}{6}\,(1)^2 = \frac{1}{2}$

12. (a) Let $u = 2 + \tan\frac{t}{2} \Rightarrow du = \frac{1}{2}\sec^2\frac{t}{2}\,dt \Rightarrow 2\,du = \sec^2\frac{t}{2}\,dt$; $t = \frac{-\pi}{2} \Rightarrow u = 2 + \tan\left(\frac{-\pi}{4}\right) = 1$, $t = 0 \Rightarrow u = 2$

$$\int_{-\pi/2}^{0}\left(2 + \tan\frac{t}{2}\right)\sec^2\frac{t}{2}\,dt = \int_1^2 u\,(2\,du) = \left[u^2\right]_1^2 = 2^2 - 1^2 = 3$$

(b) Use the same substitution as in part (a); $t = \frac{-\pi}{2} \Rightarrow u = 1$, $t = \frac{\pi}{2} \Rightarrow u = 3$

$$\int_{-\pi/2}^{\pi/2}\left(2 + \tan\frac{t}{2}\right)\sec^2\frac{t}{2}\,dt = 2\int_1^3 u\,du = \left[u^2\right]_1^3 = 3^2 - 1^2 = 8$$

13. (a) Let $u = 4 + 3\sin z \Rightarrow du = 3\cos z\,dz \Rightarrow \frac{1}{3}\,du = \cos z\,dz$; $z = 0 \Rightarrow u = 4$, $z = 2\pi \Rightarrow u = 4$

$$\int_0^{2\pi}\frac{\cos z}{\sqrt{4 + 3\sin z}}\,dz = \int_4^4 \frac{1}{\sqrt{u}}\left(\frac{1}{3}\,du\right) = 0$$

(b) Use the same substitution as in part (a); $z = -\pi \Rightarrow u = 4 + 3\sin(-\pi) = 4$, $z = \pi \Rightarrow u = 4$

$$\int_{-\pi}^{\pi}\frac{\cos z}{\sqrt{4 + 3\sin z}}\,dz = \int_4^4 \frac{1}{\sqrt{u}}\left(\frac{1}{3}\,du\right) = 0$$

14. (a) Let $u = 3 + 2\cos w \Rightarrow du = -2\sin w\,dw \Rightarrow -\frac{1}{2}\,du = \sin w\,dw$; $w = -\frac{\pi}{2} \Rightarrow u = 3$, $w = 0 \Rightarrow u = 5$

$$\int_{-\pi/2}^{0}\frac{\sin w}{(3 + 2\cos w)^2}\,dw = \int_3^5 u^{-2}\left(-\frac{1}{2}\,du\right) = \frac{1}{2}\left[u^{-1}\right]_3^5 = \frac{1}{2}\left(\frac{1}{5} - \frac{1}{3}\right) = -\frac{1}{15}$$

(b) Use the same substitution as in part (a); $w = 0 \Rightarrow u = 5$, $w = \frac{\pi}{2} \Rightarrow u = 3$

$$\int_0^{\pi/2}\frac{\sin w}{(3 + 2\cos w)^2}\,dw = \int_5^3 u^{-2}\left(-\frac{1}{2}\,du\right) = \frac{1}{2}\int_3^5 u^{-2}\,du = \frac{1}{15}$$

15. Let $u = t^5 + 2t \Rightarrow du = (5t^4 + 2)\,dt$; $t = 0 \Rightarrow u = 0$, $t = 1 \Rightarrow u = 3$

$$\int_0^1 \sqrt{t^5 + 2t}\,(5t^4 + 2)\,dt = \int_0^3 u^{1/2}\,du = \left[\frac{2}{3}u^{3/2}\right]_0^3 = \frac{2}{3}(3)^{3/2} - \frac{2}{3}(0)^{3/2} = 2\sqrt{3}$$

16. Let $u = 1 + \sqrt{y} \Rightarrow du = \frac{dy}{2\sqrt{y}}$; $y = 1 \Rightarrow u = 2$, $y = 4 \Rightarrow u = 3$

$$\int_1^4 \frac{dy}{2\sqrt{y}\left(1 + \sqrt{y}\right)^2} = \int_2^3 \frac{1}{u^2}\,du = \int_2^3 u^{-2}\,du = \left[-u^{-1}\right]_2^3 = \left(-\frac{1}{3}\right) - \left(-\frac{1}{2}\right) = \frac{1}{6}$$

17. Let $u = \cos 2\theta \Rightarrow du = -2\sin 2\theta\,d\theta \Rightarrow -\frac{1}{2}\,du = \sin 2\theta\,d\theta$; $\theta = 0 \Rightarrow u = 1$, $\theta = \frac{\pi}{6} \Rightarrow u = \cos 2\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$$\int_0^{\pi/6}\cos^{-3}2\theta\,\sin 2\theta\,d\theta = \int_1^{1/2} u^{-3}\left(-\frac{1}{2}\,du\right) = -\frac{1}{2}\int_1^{1/2} u^{-3}\,du = \left[-\frac{1}{2}\left(\frac{u^{-2}}{-2}\right)\right]_1^{1/2} = \frac{1}{4\left(\frac{1}{2}\right)^2} - \frac{1}{4(1)^2} = \frac{3}{4}$$

18. Let $u = \tan\left(\frac{\theta}{6}\right) \Rightarrow du = \frac{1}{6}\sec^2\left(\frac{\theta}{6}\right)d\theta \Rightarrow 6\,du = \sec^2\left(\frac{\theta}{6}\right)d\theta$; $\theta = \pi \Rightarrow u = \tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$, $\theta = \frac{3\pi}{2} \Rightarrow$ $u = \tan\frac{\pi}{4} = 1$

$$\int_{\pi}^{3\pi/2}\cot^5\left(\frac{\theta}{6}\right)\sec^2\left(\frac{\theta}{6}\right)d\theta = \int_{1/\sqrt{3}}^{1} u^{-5}\,(6\,du) = \left[6\left(\frac{u^{-4}}{-4}\right)\right]_{1/\sqrt{3}}^{1} = \left[-\frac{3}{2u^4}\right]_{1/\sqrt{3}}^{1} = -\frac{3}{2(1)^4} - \left(-\frac{3}{2\left(\frac{1}{\sqrt{3}}\right)^4}\right) = 12$$

19. Let $u = 5 - 4\cos t \Rightarrow du = 4\sin t\,dt \Rightarrow \frac{1}{4}\,du = \sin t\,dt$; $t = 0 \Rightarrow u = 5 - 4\cos 0 = 1$, $t = \pi \Rightarrow$ $u = 5 - 4\cos\pi = 9$

$$\int_0^{\pi} 5\,(5 - 4\cos t)^{1/4}\sin t\,dt = \int_1^9 5u^{1/4}\left(\frac{1}{4}\,du\right) = \frac{5}{4}\int_1^9 u^{1/4}\,du = \left[\frac{5}{4}\left(\frac{4}{5}u^{5/4}\right)\right]_1^9 = 9^{5/4} - 1 = 3^{5/2} - 1$$

20. Let $u = 1 - \sin 2t \Rightarrow du = -2\cos 2t\,dt \Rightarrow -\frac{1}{2}\,du = \cos 2t\,dt$; $t = 0 \Rightarrow u = 1$, $t = \frac{\pi}{4} \Rightarrow u = 0$

$$\int_0^{\pi/4}(1 - \sin 2t)^{3/2}\cos 2t\,dt = \int_1^0 -\frac{1}{2}u^{3/2}\,du = \left[-\frac{1}{2}\left(\frac{2}{5}u^{5/2}\right)\right]_1^0 = \left(-\frac{1}{5}(0)^{5/2}\right) - \left(-\frac{1}{5}(1)^{5/2}\right) = \frac{1}{5}$$

21. Let $u = 4y - y^2 + 4y^3 + 1 \Rightarrow du = (4 - 2y + 12y^2)\,dy$; $y = 0 \Rightarrow u = 1$, $y = 1 \Rightarrow u = 4(1) - (1)^2 + 4(1)^3 + 1 = 8$

$$\int_0^1 (4y - y^2 + 4y^3 + 1)^{-2/3}(12y^2 - 2y + 4)\,dy = \int_1^8 u^{-2/3}\,du = \left[3u^{1/3}\right]_1^8 = 3(8)^{1/3} - 3(1)^{1/3} = 3$$

22. Let $u = y^3 + 6y^2 - 12y + 9 \Rightarrow du = (3y^2 + 12y - 12)\,dy \Rightarrow \frac{1}{3}\,du = (y^2 + 4y - 4)\,dy$; $y = 0 \Rightarrow u = 9$, $y = 1 \Rightarrow u = 4$

$\int_0^1 (y^3 + 6y^2 - 12y + 9)^{-1/2}(y^2 + 4y - 4)\,dy = \int_9^4 \frac{1}{3}u^{-1/2}\,du = \left[\frac{1}{3}\left(2u^{1/2}\right)\right]_9^4 = \frac{2}{3}(4)^{1/2} - \frac{2}{3}(9)^{1/2} = \frac{2}{3}(2-3) = -\frac{2}{3}$

23. Let $u = \theta^{3/2} \Rightarrow du = \frac{3}{2}\theta^{1/2}\,d\theta \Rightarrow \frac{2}{3}\,du = \sqrt{\theta}\,d\theta$; $\theta = 0 \Rightarrow u = 0$, $\theta = \sqrt[3]{\pi^2} \Rightarrow u = \pi$

$\int_0^{\sqrt[3]{\pi^2}} \sqrt{\theta}\cos^2\left(\theta^{3/2}\right)d\theta = \int_0^{\pi}\cos^2 u\left(\frac{2}{3}\,du\right) = \left[\frac{2}{3}\left(\frac{u}{2} + \frac{1}{4}\sin 2u\right)\right]_0^{\pi} = \frac{2}{3}\left(\frac{\pi}{2} + \frac{1}{4}\sin 2\pi\right) - \frac{2}{3}(0) = \frac{\pi}{3}$

24. Let $u = 1 + \frac{1}{t} \Rightarrow du = -t^{-2}\,dt$; $t = -1 \Rightarrow u = 0$, $t = -\frac{1}{2} \Rightarrow u = -1$

$\int_{-1}^{-1/2} t^{-2}\sin^2\left(1 + \frac{1}{t}\right)dt = \int_0^{-1} -\sin^2 u\,du = \left[-\left(\frac{u}{2} - \frac{1}{4}\sin 2u\right)\right]_0^{-1} = -\left[\left(-\frac{1}{2} - \frac{1}{4}\sin(-2)\right) - \left(\frac{0}{2} - \frac{1}{4}\sin 0\right)\right] = \frac{1}{2} - \frac{1}{4}\sin 2$

25. Let $u = 4 - x^2 \Rightarrow du = -2x\,dx \Rightarrow -\frac{1}{2}\,du = x\,dx$; $x = -2 \Rightarrow u = 0$, $x = 0 \Rightarrow u = 4$, $x = 2 \Rightarrow u = 0$

$A = -\int_{-2}^{0} x\sqrt{4 - x^2}\,dx + \int_0^2 x\sqrt{4 - x^2}\,dx = -\int_0^4 -\frac{1}{2}u^{1/2}\,du + \int_4^0 -\frac{1}{2}u^{1/2}\,du = 2\int_0^4 \frac{1}{2}u^{1/2}\,du = \int_0^4 u^{1/2}\,du = \left[\frac{2}{3}u^{3/2}\right]_0^4 = \frac{2}{3}(4)^{3/2} - \frac{2}{3}(0)^{3/2} = \frac{16}{3}$

26. Let $u = 1 - \cos x \Rightarrow du = \sin x\,dx$; $x = 0 \Rightarrow u = 0$, $x = \pi \Rightarrow u = 2$

$\int_0^{\pi}(1 - \cos x)\sin x\,dx = \int_0^2 u\,du = \left[\frac{u^2}{2}\right]_0^2 = \frac{2^2}{2} - \frac{0^2}{2} = 2$

27. Let $u = 1 + \cos x \Rightarrow du = -\sin x\,dx \Rightarrow -du = \sin x\,dx$; $x = -\pi \Rightarrow u = 1 + \cos(-\pi) = 0$, $x = 0 \Rightarrow u = 1 + \cos 0 = 2$

$A = -\int_{-\pi}^{0} 3(\sin x)\sqrt{1 + \cos x}\,dx = -\int_0^2 3u^{1/2}(-du) = 3\int_0^2 u^{1/2}\,du = \left[2u^{3/2}\right]_0^2 = 2(2)^{3/2} - 2(0)^{3/2} = 2^{5/2}$

28. Let $u = \pi + \pi\sin x \Rightarrow du = \pi\cos x\,dx \Rightarrow \frac{1}{\pi}\,du = \cos x\,dx$; $x = -\frac{\pi}{2} \Rightarrow u = \pi + \pi\sin\left(-\frac{\pi}{2}\right) = 0$, $x = 0 \Rightarrow u = \pi$

Because of symmetry about $x = -\frac{\pi}{2}$, $A = 2\int_{-\pi/2}^{0} \frac{\pi}{2}(\cos x)(\sin(\pi + \pi\sin x))\,dx = 2\int_0^{\pi}\frac{\pi}{2}(\sin u)\left(\frac{1}{\pi}\,du\right) = \int_0^{\pi}\sin u\,du = [-\cos u]_0^{\pi} = (-\cos\pi) - (-\cos 0) = 2$

29. For the sketch given, $a = 0$, $b = \pi$; $f(x) - g(x) = 1 - \cos^2 x = \sin^2 x = \frac{1 - \cos 2x}{2}$;

$A = \int_0^{\pi}\frac{(1 - \cos 2x)}{2}\,dx = \frac{1}{2}\int_0^{\pi}(1 - \cos 2x)\,dx = \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\pi} = \frac{1}{2}[(\pi - 0) - (0 - 0)] = \frac{\pi}{2}$

30. For the sketch given, $a = -\frac{\pi}{3}$, $b = \frac{\pi}{3}$; $f(t) - g(t) = \frac{1}{2}\sec^2 t - (-4\sin^2 t) = \frac{1}{2}\sec^2 t + 4\sin^2 t$;

$A = \int_{-\pi/3}^{\pi/3}\left(\frac{1}{2}\sec^2 t + 4\sin^2 t\right)dt = \frac{1}{2}\int_{-\pi/3}^{\pi/3}\sec^2 t\,dt + 4\int_{-\pi/3}^{\pi/3}\sin^2 t\,dt = \frac{1}{2}\int_{-\pi/3}^{\pi/3}\sec^2 t\,dt + 4\int_{-\pi/3}^{\pi/3}\frac{(1 - \cos 2t)}{2}\,dt = \frac{1}{2}\int_{-\pi/3}^{\pi/3}\sec^2 t\,dt + 2\int_{-\pi/3}^{\pi/3}(1 - \cos 2t)\,dt = \frac{1}{2}[\tan t]_{-\pi/3}^{\pi/3} + 2\left[t - \frac{\sin 2t}{2}\right]_{-\pi/3}^{\pi/3} = \sqrt{3} + 4\cdot\frac{\pi}{3} - \sqrt{3} = \frac{4\pi}{3}$

31. For the sketch given, $a = -2$, $b = 2$; $f(x) - g(x) = 2x^2 - (x^4 - 2x^2) = 4x^2 - x^4$;

$A = \int_{-2}^{2}(4x^2 - x^4)\,dx = \left[\frac{4x^3}{3} - \frac{x^5}{5}\right]_{-2}^{2} = \left(\frac{32}{3} - \frac{32}{5}\right) - \left[-\frac{32}{3} - \left(-\frac{32}{5}\right)\right] = \frac{64}{3} - \frac{64}{5} = \frac{320-192}{15} = \frac{128}{15}$

32. For the sketch given, $c = 0$, $d = 1$; $f(y) - g(y) = y^2 - y^3$;
$A = \int_0^1 (y^2 - y^3)\, dy = \int_0^1 y^2\, dy - \int_0^1 y^3\, dy = \left[\frac{y^3}{3}\right]_0^1 - \left[\frac{y^4}{4}\right]_0^1 = \frac{(1-0)}{3} - \frac{(1-0)}{4} = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$

33. For the sketch given, $c = 0$, $d = 1$; $f(y) - g(y) = (12y^2 - 12y^3) - (2y^2 - 2y) = 10y^2 - 12y^3 + 2y$;
$A = \int_0^1 (10y^2 - 12y^3 + 2y)\, dy = \int_0^1 10y^2\, dy - \int_0^1 12y^3\, dy + \int_0^1 2y\, dy = \left[\frac{10}{3} y^3\right]_0^1 - \left[\frac{12}{4} y^4\right]_0^1 + \left[\frac{2}{2} y^2\right]_0^1$
$= \left(\frac{10}{3} - 0\right) - (3 - 0) + (1 - 0) = \frac{4}{3}$

34. For the sketch given, $a = -1$, $b = 1$; $f(x) - g(x) = x^2 - (-2x^4) = x^2 + 2x^4$;
$A = \int_{-1}^1 (x^2 + 2x^4)\, dx = \left[\frac{x^3}{3} + \frac{2x^5}{5}\right]_{-1}^1 = \left(\frac{1}{3} + \frac{2}{5}\right) - \left[-\frac{1}{3} + \left(-\frac{2}{5}\right)\right] = \frac{2}{3} + \frac{4}{5} = \frac{10+12}{15} = \frac{22}{15}$

35. We want the area between the line $y = 1$, $0 \le x \le 2$, and the curve $y = \frac{x^2}{4}$, *minus* the area of a triangle (formed by $y = x$ and $y = 1$) with base 1 and height 1. Thus, $A = \int_0^2 \left(1 - \frac{x^2}{4}\right) dx - \frac{1}{2}(1)(1) = \left[x - \frac{x^3}{12}\right]_0^2 - \frac{1}{2}$
$= \left(2 - \frac{8}{12}\right) - \frac{1}{2} = 2 - \frac{2}{3} - \frac{1}{2} = \frac{5}{6}$

36. We want the area between the x-axis and the curve $y = x^2$, $0 \le x \le 1$ *plus* the area of a triangle (formed by $x = 1$, $x + y = 2$, and the x-axis) with base 1 and height 1. Thus, $A = \int_0^1 x^2\, dx + \frac{1}{2}(1)(1) = \left[\frac{x^3}{3}\right]_0^1 + \frac{1}{2} = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$

37. AREA = A1 + A2
A1: For the sketch given, $a = -3$ and we find b by solving the equations $y = x^2 - 4$ and $y = -x^2 - 2x$ simultaneously for x: $x^2 - 4 = -x^2 - 2x \Rightarrow 2x^2 + 2x - 4 = 0 \Rightarrow 2(x+2)(x-1) \Rightarrow x = -2$ or $x = 1$ so $b = -2$: $f(x) - g(x) = (x^2 - 4) - (-x^2 - 2x) = 2x^2 + 2x - 4 \Rightarrow A1 = \int_{-3}^{-2} (2x^2 + 2x - 4)\, dx$
$= \left[\frac{2x^3}{3} + \frac{2x^2}{2} - 4x\right]_{-3}^{-2} = \left(-\frac{16}{3} + 4 + 8\right) - (-18 + 9 + 12) = 9 - \frac{16}{3} = \frac{11}{3}$;
A2: For the sketch given, $a = -2$ and $b = 1$: $f(x) - g(x) = (-x^2 - 2x) - (x^2 - 4) = -2x^2 - 2x + 4$
$\Rightarrow A2 = -\int_{-2}^1 (2x^2 + 2x - 4)\, dx = -\left[\frac{2x^3}{3} + x^2 - 4x\right]_{-2}^1 = -\left(\frac{2}{3} + 1 - 4\right) + \left(-\frac{16}{3} + 4 + 8\right)$
$= -\frac{2}{3} - 1 + 4 - \frac{16}{3} + 4 + 8 = 9$;
Therefore, AREA = A1 + A2 $= \frac{11}{3} + 9 = \frac{38}{3}$

38. AREA = A1 + A2
A1: For the sketch given, $a = -2$ and $b = 0$: $f(x) - g(x) = (2x^3 - x^2 - 5x) - (-x^2 + 3x) = 2x^3 - 8x$
$\Rightarrow A1 = \int_{-2}^0 (2x^3 - 8x)\, dx = \left[\frac{2x^4}{4} - \frac{8x^2}{2}\right]_{-2}^0 = 0 - (8 - 16) = 8$;
A2: For the sketch given, $a = 0$ and $b = 2$: $f(x) - g(x) = (-x^2 + 3x) - (2x^3 - x^2 - 5x) = 8x - 2x^3$
$\Rightarrow A2 = \int_0^2 (8x - 2x^3)\, dx = \left[\frac{8x^2}{2} - \frac{2x^4}{4}\right]_0^2 = (16 - 8) = 8$;
Therefore, AREA = A1 + A2 = 16

39. AREA = A1 + A2 + A3
A1: For the sketch given, $a = -2$ and $b = -1$: $f(x) - g(x) = (-x + 2) - (4 - x^2) = x^2 - x - 2$
$\Rightarrow A1 = \int_{-2}^{-1} (x^2 - x - 2)\, dx = \left[\frac{x^3}{3} - \frac{x^2}{2} - 2x\right]_{-2}^{-1} = \left(-\frac{1}{3} - \frac{1}{2} + 2\right) - \left(-\frac{8}{3} - \frac{4}{2} + 4\right) = \frac{7}{3} - \frac{1}{2} = \frac{14-3}{6} = \frac{11}{6}$;
A2: For the sketch given, $a = -1$ and $b = 2$: $f(x) - g(x) = (4 - x^2) - (-x + 2) = -(x^2 - x - 2)$
$\Rightarrow A2 = -\int_{-1}^2 (x^2 - x - 2)\, dx = -\left[\frac{x^3}{3} - \frac{x^2}{2} - 2x\right]_{-1}^2 = -\left(\frac{8}{3} - \frac{4}{2} - 4\right) + \left(-\frac{1}{3} - \frac{1}{2} + 2\right) = -3 + 8 - \frac{1}{2} = \frac{9}{2}$;

A3: For the sketch given, $a = 2$ and $b = 3$: $f(x) - g(x) = (-x + 2) - (4 - x^2) = x^2 - x - 2$

$$\Rightarrow A3 = \int_2^3 (x^2 - x - 2)\,dx = \left[\frac{x^3}{3} - \frac{x^2}{2} - 2x\right]_2^3 = \left(\frac{27}{3} - \frac{9}{2} - 6\right) - \left(\frac{8}{3} - \frac{4}{2} - 4\right) = 9 - \frac{9}{2} - \frac{8}{3};$$

Therefore, $\text{AREA} = A1 + A2 + A3 = \frac{11}{6} + \frac{9}{2} + \left(9 - \frac{9}{2} - \frac{8}{3}\right) = 9 - \frac{5}{6} = \frac{49}{6}$

40. AREA = A1 + A2 + A3

A1: For the sketch given, $a = -2$ and $b = 0$: $f(x) - g(x) = \left(\frac{x^3}{3} - x\right) - \frac{x}{3} = \frac{x^3}{3} - \frac{4}{3}x = \frac{1}{3}(x^3 - 4x)$

$$\Rightarrow A1 = \frac{1}{3}\int_{-2}^0 (x^3 - 4x)\,dx = \frac{1}{3}\left[\frac{x^4}{4} - 2x^2\right]_{-2}^0 = 0 - \frac{1}{3}(4 - 8) = \frac{4}{3};$$

A2: For the sketch given, $a = 0$ and we find b by solving the equations $y = \frac{x^3}{3} - x$ and $y = \frac{x}{3}$ simultaneously for x: $\frac{x^3}{3} - x = \frac{x}{3} \Rightarrow \frac{x^3}{3} - \frac{4}{3}x = 0 \Rightarrow \frac{x}{3}(x - 2)(x + 2) = 0 \Rightarrow x = -2, x = 0$, or $x = 2$ so $b = 2$:

$$f(x) - g(x) = \frac{x}{3} - \left(\frac{x^3}{3} - x\right) = -\frac{1}{3}(x^3 - 4x) \Rightarrow A2 = -\frac{1}{3}\int_0^2 (x^3 - 4x)\,dx = \frac{1}{3}\int_0^2 (4x - x^3) = \frac{1}{3}\left[2x^2 - \frac{x^4}{4}\right]_0^2$$
$$= \frac{1}{3}(8 - 4) = \frac{4}{3};$$

A3: For the sketch given, $a = 2$ and $b = 3$: $f(x) - g(x) = \left(\frac{x^3}{3} - x\right) - \frac{x}{3} = \frac{1}{3}(x^3 - 4x)$

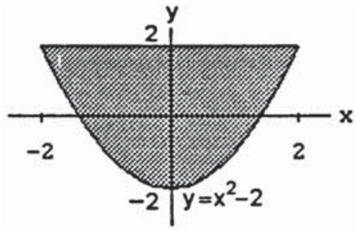
$$\Rightarrow A3 = \frac{1}{3}\int_2^3 (x^3 - 4x)\,dx = \frac{1}{3}\left[\frac{x^4}{4} - 2x^2\right]_2^3 = \frac{1}{3}\left[\left(\frac{81}{4} - 2\cdot 9\right) - \left(\frac{16}{4} - 8\right)\right] = \frac{1}{3}\left(\frac{81}{4} - 14\right) = \frac{25}{12};$$

Therefore, $\text{AREA} = A1 + A2 + A3 = \frac{4}{3} + \frac{4}{3} + \frac{25}{12} = \frac{32+25}{12} = \frac{19}{4}$

41. $a = -2, b = 2$;

$f(x) - g(x) = 2 - (x^2 - 2) = 4 - x^2$

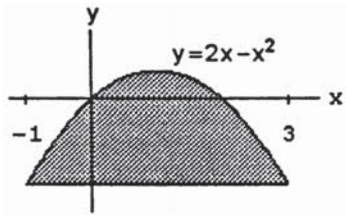
$$\Rightarrow A = \int_{-2}^2 (4 - x^2)dx = \left[4x - \frac{x^3}{3}\right]_{-2}^2 = \left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right)$$
$$= 2\cdot\left(\frac{24}{3} - \frac{8}{3}\right) = \frac{32}{3}$$



42. $a = -1, b = 3$;

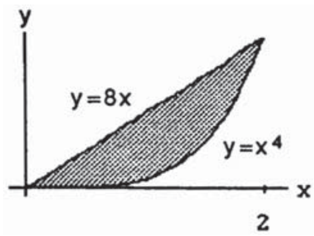
$f(x) - g(x) = (2x - x^2) - (-3) = 2x - x^2 + 3$

$$\Rightarrow A = \int_{-1}^3 (2x - x^2 + 3)\,dx = \left[x^2 - \frac{x^3}{3} + 3x\right]_{-1}^3$$
$$= \left(9 - \frac{27}{3} + 9\right) - \left(1 + \frac{1}{3} - 3\right) = 11 - \frac{1}{3} = \frac{32}{3}$$



43. $a = 0, b = 2$;

$$f(x) - g(x) = 8x - x^4 \Rightarrow A = \int_0^2 (8x - x^4)\,dx$$
$$= \left[\frac{8x^2}{2} - \frac{x^5}{5}\right]_0^2 = 16 - \frac{32}{5} = \frac{80-32}{5} = \frac{48}{5}$$

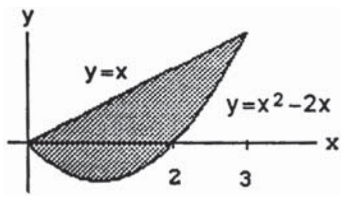


44. Limits of integration: $x^2 - 2x = x \Rightarrow x^2 = 3x$

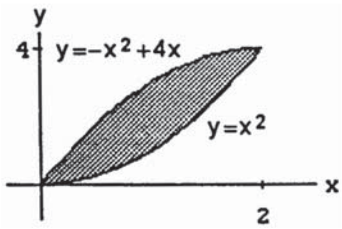
$\Rightarrow x(x - 3) = 0 \Rightarrow a = 0$ and $b = 3$;

$f(x) - g(x) = x - (x^2 - 2x) = 3x - x^2$

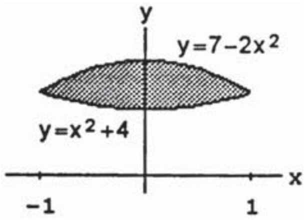
$$\Rightarrow A = \int_0^3 (3x - x^2)\,dx = \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3$$
$$= \frac{27}{2} - 9 = \frac{27-18}{2} = \frac{9}{2}$$



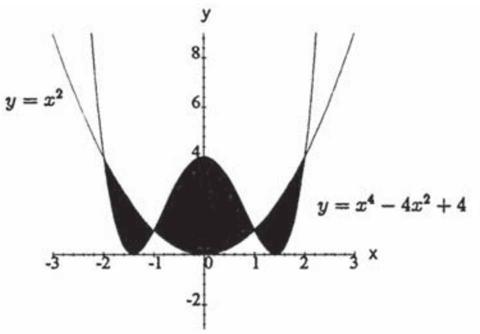
45. Limits of integration: $x^2 = -x^2 + 4x \Rightarrow 2x^2 - 4x = 0$
$\Rightarrow 2x(x-2) = 0 \Rightarrow a = 0$ and $b = 2$;
$f(x) - g(x) = (-x^2 + 4x) - x^2 = -2x^2 + 4x$
$\Rightarrow A = \int_0^2 (-2x^2 + 4x)\,dx = \left[\frac{-2x^3}{3} + \frac{4x^2}{2}\right]_0^2$
$= -\frac{16}{3} + \frac{16}{2} = \frac{-32+48}{6} = \frac{8}{3}$



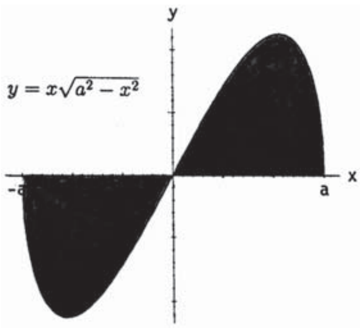
46. Limits of integration: $7 - 2x^2 = x^2 + 4 \Rightarrow 3x^2 - 3 = 0$
$\Rightarrow 3(x-1)(x+1) = 0 \Rightarrow a = -1$ and $b = 1$;
$f(x) - g(x) = (7 - 2x^2) - (x^2 + 4) = 3 - 3x^2$
$\Rightarrow A = \int_{-1}^1 (3 - 3x^2)\,dx = 3\left[x - \frac{x^3}{3}\right]_{-1}^1$
$= 3\left[\left(1 - \frac{1}{3}\right) - \left(-1 + \frac{1}{3}\right)\right] = 6\left(\frac{2}{3}\right) = 4$



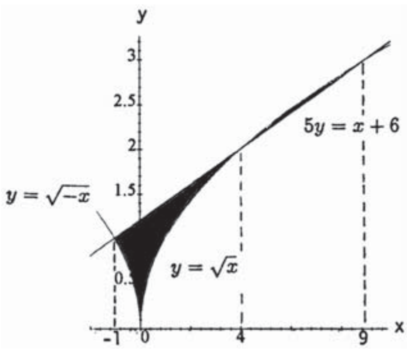
47. Limits of integration: $x^4 - 4x^2 + 4 = x^2$
$\Rightarrow x^4 - 5x^2 + 4 = 0 \Rightarrow (x^2 - 4)(x^2 - 1) = 0$
$\Rightarrow (x+2)(x-2)(x+1)(x-1) = 0 \Rightarrow x = -2, -1, 1, 2$;
$f(x) - g(x) = (x^4 - 4x^2 + 4) - x^2 = x^4 - 5x^2 + 4$ and
$g(x) - f(x) = x^2 - (x^4 - 4x^2 + 4) = -x^4 + 5x^2 - 4$
$\Rightarrow A = \int_{-2}^{-1} (-x^4 + 5x^2 - 4)dx + \int_{-1}^1 (x^4 - 5x^2 + 4)dx$
$+ \int_1^2 (-x^4 + 5x^2 - 4)dx$
$= \left[-\frac{x^5}{5} + \frac{5x^3}{3} - 4x\right]_{-2}^{-1} + \left[\frac{x^5}{5} - \frac{5x^3}{3} + 4x\right]_{-1}^1 + \left[\frac{-x^5}{5} + \frac{5x^3}{3} - 4x\right]_1^2$
$= \left(\frac{1}{5} - \frac{5}{3} + 4\right) - \left(\frac{32}{5} - \frac{40}{3} + 8\right) + \left(\frac{1}{5} - \frac{5}{3} + 4\right) - \left(-\frac{1}{5} + \frac{5}{3} - 4\right) + \left(-\frac{32}{5} + \frac{40}{3} - 8\right) - \left(-\frac{1}{5} + \frac{5}{3} - 4\right)$
$= -\frac{60}{5} + \frac{60}{3} = \frac{300-180}{15} = 8$



48. Limits of integration: $x\sqrt{a^2 - x^2} = 0 \Rightarrow x = 0$ or
$\sqrt{a^2 - x^2} = 0 \Rightarrow x = 0$ or $a^2 - x^2 = 0 \Rightarrow x = -a, 0, a$;
$A = \int_{-a}^0 -x\sqrt{a^2 - x^2}\,dx + \int_0^a x\sqrt{a^2 - x^2}\,dx$
$= \frac{1}{2}\left[\frac{2}{3}(a^2 - x^2)^{3/2}\right]_{-a}^0 - \frac{1}{2}\left[\frac{2}{3}(a^2 - x^2)^{3/2}\right]_0^a$
$= \frac{1}{3}(a^2)^{3/2} - \left[-\frac{1}{3}(a^2)^{3/2}\right] = \frac{2a^3}{3}$



49. Limits of integration: $y = \sqrt{|x|} = \begin{cases} \sqrt{-x}, & x \le 0 \\ \sqrt{x}, & x \ge 0 \end{cases}$ and
$5y = x + 6$ or $y = \frac{x}{5} + \frac{6}{5}$; for $x \le 0$: $\sqrt{-x} = \frac{x}{5} + \frac{6}{5}$
$\Rightarrow 5\sqrt{-x} = x + 6 \Rightarrow 25(-x) = x^2 + 12x + 36$
$\Rightarrow x^2 + 37x + 36 = 0 \Rightarrow (x+1)(x+36) = 0$
$\Rightarrow x = -1, -36$ (but $x = -36$ is not a solution);
for $x \ge 0$: $5\sqrt{x} = x + 6 \Rightarrow 25x = x^2 + 12x + 36$
$\Rightarrow x^2 - 13x + 36 = 0 \Rightarrow (x-4)(x-9) = 0$
$\Rightarrow x = 4, 9$; there are three intersection points and



$A = \int_{-1}^0 \left(\frac{x+6}{5} - \sqrt{-x}\right)dx + \int_0^4 \left(\frac{x+6}{5} - \sqrt{x}\right)dx + \int_4^9 \left(\sqrt{x} - \frac{x+6}{5}\right)dx$

$$= \left[\frac{(x+6)^2}{10} + \frac{2}{3}(-x)^{3/2}\right]_{-1}^{0} + \left[\frac{(x+6)^2}{10} - \frac{2}{3}x^{3/2}\right]_{0}^{4} + \left[\frac{2}{3}x^{3/2} - \frac{(x+6)^2}{10}\right]_{4}^{9}$$

$$= \left(\frac{36}{10} - \frac{25}{10} - \frac{2}{3}\right) + \left(\frac{100}{10} - \frac{2}{3}\cdot 4^{3/2} - \frac{36}{10} + 0\right) + \left(\frac{2}{3}\cdot 9^{3/2} - \frac{225}{10} - \frac{2}{3}\cdot 4^{3/2} + \frac{100}{10}\right) = -\frac{50}{10} + \frac{20}{3} = \frac{5}{3}$$

50. Limits of integration:

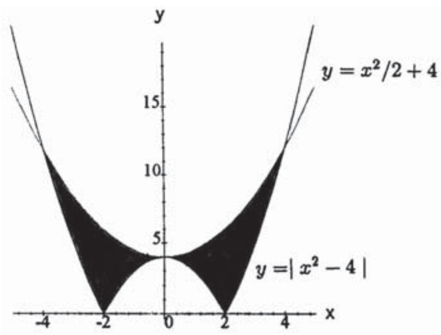
$y = |x^2 - 4| = \begin{cases} x^2 - 4, & x \le -2 \text{ or } x \ge 2 \\ 4 - x^2, & -2 \le x \le 2 \end{cases}$

for $x \le -2$ and $x \ge 2$: $x^2 - 4 = \frac{x^2}{2} + 4$

$\Rightarrow 2x^2 - 8 = x^2 + 8 \Rightarrow x^2 = 16 \Rightarrow x = \pm 4$;

for $-2 \le x \le 2$: $4 - x^2 = \frac{x^2}{2} + 4 \Rightarrow 8 - 2x^2 = x^2 + 8$

$\Rightarrow x^2 = 0 \Rightarrow x = 0$; by symmetry of the graph,



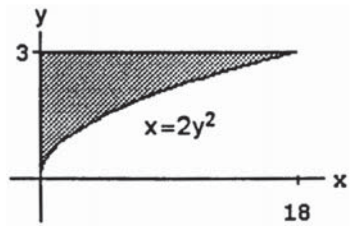
$$A = 2\int_0^2 \left[\left(\frac{x^2}{2} + 4\right) - (4 - x^2)\right] dx + 2\int_2^4 \left[\left(\frac{x^2}{2} + 4\right) - (x^2 - 4)\right] dx = 2\left[\frac{x^3}{2}\right]_0^2 + 2\left[8x - \frac{x^3}{6}\right]_2^4$$

$$= 2\left(\frac{8}{2} - 0\right) + 2\left(32 - \frac{64}{6} - 16 + \frac{8}{6}\right) = 40 - \frac{56}{3} = \frac{64}{3}$$

51. Limits of integration: $c = 0$ and $d = 3$;

$f(y) - g(y) = 2y^2 - 0 = 2y^2$

$\Rightarrow A = \int_0^3 2y^2\,dy = \left[\frac{2y^3}{3}\right]_0^3 = 2 \cdot 9 = 18$

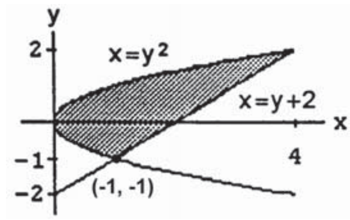


52. Limits of integration: $y^2 = y + 2 \Rightarrow (y+1)(y-2) = 0$

$\Rightarrow c = -1$ and $d = 2$; $f(y) - g(y) = (y + 2) - y^2$

$\Rightarrow A = \int_{-1}^{2} (y + 2 - y^2)\,dy = \left[\frac{y^2}{2} + 2y - \frac{y^3}{3}\right]_{-1}^{2}$

$= \left(\frac{4}{2} + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right) = 6 - \frac{8}{3} - \frac{1}{2} + 2 - \frac{1}{3} = \frac{9}{2}$



53. Limits of integration: $4x = y^2 - 4$ and $4x = 16 + y$

$\Rightarrow y^2 - 4 = 16 + y \Rightarrow y^2 - y - 20 = 0 \Rightarrow$

$(y - 5)(y + 4) = 0 \Rightarrow c = -4$ and $d = 5$;

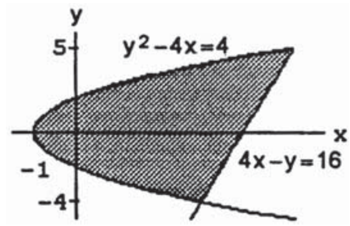
$f(y) - g(y) = \left(\frac{16+y}{4}\right) - \left(\frac{y^2-4}{4}\right) = \frac{-y^2+y+20}{4}$

$\Rightarrow A = \frac{1}{4}\int_{-4}^{5} (-y^2 + y + 20)\,dy$

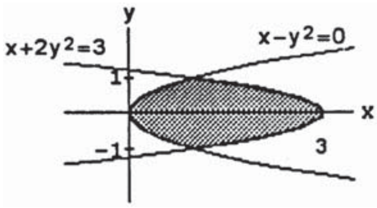
$= \frac{1}{4}\left[-\frac{y^3}{3} + \frac{y^2}{2} + 20y\right]_{-4}^{5}$

$= \frac{1}{4}\left(-\frac{125}{3} + \frac{25}{2} + 100\right) - \frac{1}{4}\left(\frac{64}{3} + \frac{16}{2} - 80\right)$

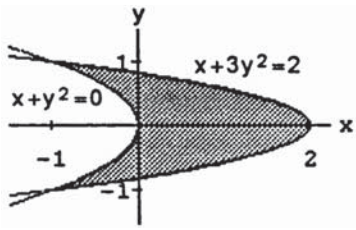
$= \frac{1}{4}\left(-\frac{189}{3} + \frac{9}{2} + 180\right) = \frac{243}{8}$



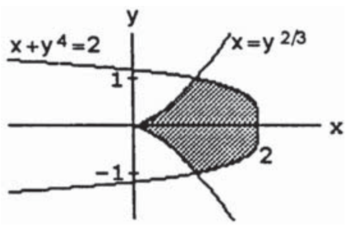
54. Limits of integration: $x = y^2$ and $x = 3 - 2y^2$
$\Rightarrow y^2 = 3 - 2y^2 \Rightarrow 3y^2 = 3 \Rightarrow 3(y-1)(y+1) = 0$
$\Rightarrow c = -1$ and $d = 1$; $f(y) - g(y) = (3 - 2y^2) - y^2$
$= 3 - 3y^2 = 3(1 - y^2) \Rightarrow A = 3\int_{-1}^{1} (1 - y^2)\, dy$
$= 3\left[y - \frac{y^3}{3}\right]_{-1}^{1} = 3\left(1 - \frac{1}{3}\right) - 3\left(-1 + \frac{1}{3}\right)$
$= 3 \cdot 2\left(1 - \frac{1}{3}\right) = 4$



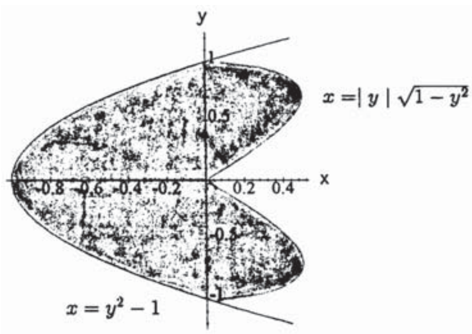
55. Limits of integration: $x = -y^2$ and $x = 2 - 3y^2$
$\Rightarrow -y^2 = 2 - 3y^2 \Rightarrow 2y^2 - 2 = 0$
$\Rightarrow 2(y-1)(y+1) = 0 \Rightarrow c = -1$ and $d = 1$;
$f(y) - g(y) = (2 - 3y^2) - (-y^2) = 2 - 2y^2 = 2(1 - y^2)$
$\Rightarrow A = 2\int_{-1}^{1} (1 - y^2)\, dy = 2\left[y - \frac{y^3}{3}\right]_{-1}^{1}$
$= 2\left(1 - \frac{1}{3}\right) - 2\left(-1 + \frac{1}{3}\right) = 4\left(\frac{2}{3}\right) = \frac{8}{3}$



56. Limits of integration: $x = y^{2/3}$ and $x = 2 - y^4$
$\Rightarrow y^{2/3} = 2 - y^4 \Rightarrow c = -1$ and $d = 1$;
$f(y) - g(y) = (2 - y^4) - y^{2/3}$
$\Rightarrow A = \int_{-1}^{1} \left(2 - y^4 - y^{2/3}\right) dy$
$= \left[2y - \frac{y^5}{5} - \frac{3}{5}y^{5/3}\right]_{-1}^{1}$
$= \left(2 - \frac{1}{5} - \frac{3}{5}\right) - \left(-2 + \frac{1}{5} + \frac{3}{5}\right)$
$= 2\left(2 - \frac{1}{5} - \frac{3}{5}\right) = \frac{12}{5}$

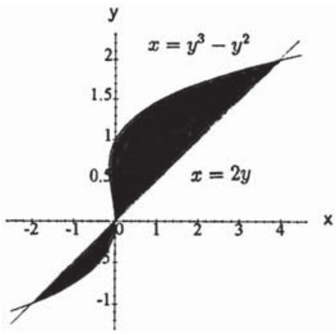


57. Limits of integration: $x = y^2 - 1$ and $x = |y|\sqrt{1 - y^2}$
$\Rightarrow y^2 - 1 = |y|\sqrt{1 - y^2} \Rightarrow y^4 - 2y^2 + 1 = y^2(1 - y^2)$
$\Rightarrow y^4 - 2y^2 + 1 = y^2 - y^4 \Rightarrow 2y^4 - 3y^2 + 1 = 0$
$\Rightarrow (2y^2 - 1)(y^2 - 1) = 0 \Rightarrow 2y^2 - 1 = 0$ or $y^2 - 1 = 0$
$\Rightarrow y^2 = \frac{1}{2}$ or $y^2 = 1 \Rightarrow y = \pm\frac{\sqrt{2}}{2}$ or $y = \pm 1$.
Substitution shows that $\frac{\pm\sqrt{2}}{2}$ are not solutions $\Rightarrow y = \pm 1$;
for $-1 \le y \le 0$, $f(x) - g(x) = -y\sqrt{1 - y^2} - (y^2 - 1)$
$= 1 - y^2 - y(1 - y^2)^{1/2}$, and by symmetry of the graph,
$A = 2\int_{-1}^{0} \left[1 - y^2 - y(1 - y^2)^{1/2}\right] dy$
$= 2\int_{-1}^{0} (1 - y^2)\, dy - 2\int_{-1}^{0} y(1 - y^2)^{1/2}\, dy$
$= 2\left[y - \frac{y^3}{3}\right]_{-1}^{0} + 2\left(\frac{1}{2}\right)\left[\frac{2(1 - y^2)^{3/2}}{3}\right]_{-1}^{0} = 2\left[(0 - 0) - \left(-1 + \frac{1}{3}\right)\right] + \left(\frac{2}{3} - 0\right) = 2$

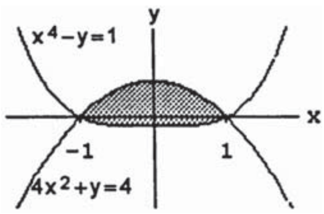


58. AREA = A1 + A2

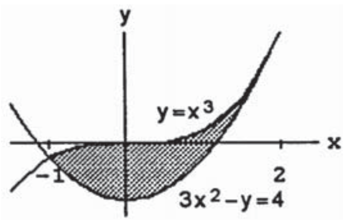
Limits of integration: $x = 2y$ and $x = y^3 - y^2 \Rightarrow$
$y^3 - y^2 = 2y \Rightarrow y(y^2 - y - 2) = y(y+1)(y-2) = 0$
$\Rightarrow y = -1, 0, 2$:
for $-1 \le y \le 0$, $f(y) - g(y) = y^3 - y^2 - 2y$
$\Rightarrow A1 = \int_{-1}^{0} (y^3 - y^2 - 2y)\,dy = \left[\frac{y^4}{4} - \frac{y^3}{3} - y^2\right]_{-1}^{0}$
$= 0 - \left(\frac{1}{4} + \frac{1}{3} - 1\right) = \frac{5}{12}$;
for $0 \le y \le 2$, $f(y) - g(y) = 2y - y^3 + y^2$
$\Rightarrow A2 = \int_{0}^{2} (2y - y^3 + y^2)\,dy = \left[y^2 - \frac{y^4}{4} + \frac{y^3}{3}\right]_{0}^{2}$
$\Rightarrow \left(4 - \frac{16}{4} + \frac{8}{3}\right) - 0 = \frac{8}{3}$;
Therefore, $A1 + A2 = \frac{5}{12} + \frac{8}{3} = \frac{37}{12}$



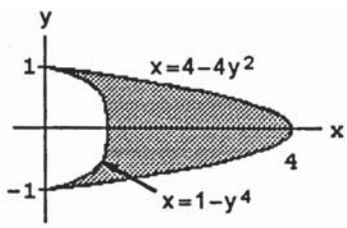
59. Limits of integration: $y = -4x^2 + 4$ and $y = x^4 - 1$
$\Rightarrow x^4 - 1 = -4x^2 + 4 \Rightarrow x^4 + 4x^2 - 5 = 0$
$\Rightarrow (x^2 + 5)(x - 1)(x + 1) = 0 \Rightarrow a = -1$ and $b = 1$;
$f(x) - g(x) = -4x^2 + 4 - x^4 + 1 = -4x^2 - x^4 + 5$
$\Rightarrow A = \int_{-1}^{1} (-4x^2 - x^4 + 5)\,dx = \left[-\frac{4x^3}{3} - \frac{x^5}{5} + 5x\right]_{-1}^{1}$
$= \left(-\frac{4}{3} - \frac{1}{5} + 5\right) - \left(\frac{4}{3} + \frac{1}{5} - 5\right) = 2\left(-\frac{4}{3} - \frac{1}{5} + 5\right) = \frac{104}{15}$



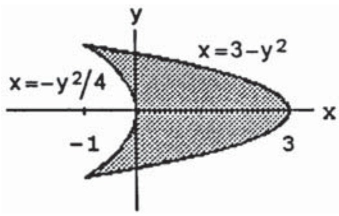
60. Limits of integration: $y = x^3$ and $y = 3x^2 - 4$
$\Rightarrow x^3 - 3x^2 + 4 = 0 \Rightarrow (x^2 - x - 2)(x - 2) = 0$
$\Rightarrow (x + 1)(x - 2)^2 = 0 \Rightarrow a = -1$ and $b = 2$;
$f(x) - g(x) = x^3 - (3x^2 - 4) = x^3 - 3x^2 + 4$
$\Rightarrow A = \int_{-1}^{2} (x^3 - 3x^2 + 4)\,dx = \left[\frac{x^4}{4} - \frac{3x^3}{3} + 4x\right]_{-1}^{2}$
$= \left(\frac{16}{4} - \frac{24}{3} + 8\right) - \left(\frac{1}{4} + 1 - 4\right) = \frac{27}{4}$



61. Limits of integration: $x = 4 - 4y^2$ and $x = 1 - y^4$
$\Rightarrow 4 - 4y^2 = 1 - y^4 \Rightarrow y^4 - 4y^2 + 3 = 0$
$\Rightarrow \left(y - \sqrt{3}\right)\left(y + \sqrt{3}\right)(y - 1)(y + 1) = 0 \Rightarrow c = -1$
and $d = 1$ since $x \ge 0$; $f(y) - g(y) = (4 - 4y^2) - (1 - y^4)$
$= 3 - 4y^2 + y^4 \Rightarrow A = \int_{-1}^{1} (3 - 4y^2 + y^4)\,dy$
$= \left[3y - \frac{4y^3}{3} + \frac{y^5}{5}\right]_{-1}^{1} = 2\left(3 - \frac{4}{3} + \frac{1}{5}\right) = \frac{56}{15}$



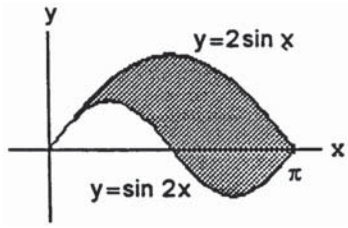
62. Limits of integration: $x = 3 - y^2$ and $x = -\frac{y^2}{4}$
$\Rightarrow 3 - y^2 = -\frac{y^2}{4} \Rightarrow \frac{3y^2}{4} - 3 = 0 \Rightarrow \frac{3}{4}(y - 2)(y + 2) = 0$
$\Rightarrow c = -2$ and $d = 2$; $f(y) - g(y) = (3 - y^2) - \left(\frac{-y^2}{4}\right)$
$= 3\left(1 - \frac{y^2}{4}\right) \Rightarrow A = 3\int_{-2}^{2} \left(1 - \frac{y^2}{4}\right) dy = 3\left[y - \frac{y^3}{12}\right]_{-2}^{2}$
$= 3\left[\left(2 - \frac{8}{12}\right) - \left(-2 + \frac{8}{12}\right)\right] = 3\left(4 - \frac{16}{12}\right) = 12 - 4 = 8$



63. $a = 0, b = \pi$; $f(x) - g(x) = 2 \sin x - \sin 2x$

$\Rightarrow A = \int_0^{\pi} (2 \sin x - \sin 2x)\, dx = \left[-2 \cos x + \frac{\cos 2x}{2}\right]_0^{\pi}$

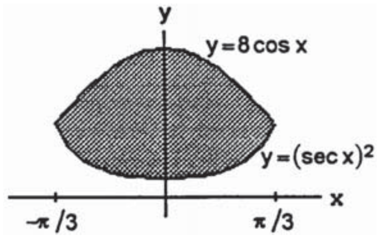
$= \left[-2(-1) + \frac{1}{2}\right] - \left(-2 \cdot 1 + \frac{1}{2}\right) = 4$



64. $a = -\frac{\pi}{3}, b = \frac{\pi}{3}$; $f(x) - g(x) = 8 \cos x - \sec^2 x$

$\Rightarrow A = \int_{-\pi/3}^{\pi/3} (8 \cos x - \sec^2 x)\, dx = [8 \sin x - \tan x]_{-\pi/3}^{\pi/3}$

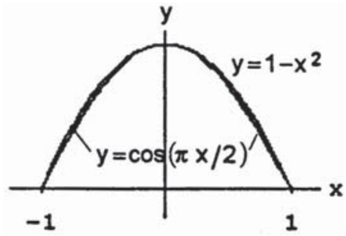
$= \left(8 \cdot \frac{\sqrt{3}}{2} - \sqrt{3}\right) - \left(-8 \cdot \frac{\sqrt{3}}{2} + \sqrt{3}\right) = 6\sqrt{3}$



65. $a = -1, b = 1$; $f(x) - g(x) = (1 - x^2) - \cos\left(\frac{\pi x}{2}\right)$

$\Rightarrow A = \int_{-1}^{1} \left[1 - x^2 - \cos\left(\frac{\pi x}{2}\right)\right] dx = \left[x - \frac{x^3}{3} - \frac{2}{\pi} \sin\left(\frac{\pi x}{2}\right)\right]_{-1}^{1}$

$= \left(1 - \frac{1}{3} - \frac{2}{\pi}\right) - \left(-1 + \frac{1}{3} + \frac{2}{\pi}\right) = 2\left(\frac{2}{3} - \frac{2}{\pi}\right) = \frac{4}{3} - \frac{4}{\pi}$



66. $A = A1 + A2$

$a_1 = -1, b_1 = 0$ and $a_2 = 0, b_2 = 1$;

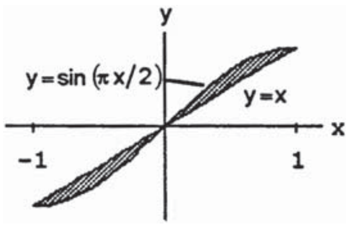
$f_1(x) - g_1(x) = x - \sin\left(\frac{\pi x}{2}\right)$ and $f_2(x) - g_2(x) = \sin\left(\frac{\pi x}{2}\right) - x$

$\Rightarrow$ by symmetry about the origin,

$A_1 + A_2 = 2A_1 \Rightarrow A = 2\int_0^1 \left[\sin\left(\frac{\pi x}{2}\right) - x\right] dx$

$= 2\left[-\frac{2}{\pi} \cos\left(\frac{\pi x}{2}\right) - \frac{x^2}{2}\right]_0^1 = 2\left[\left(-\frac{2}{\pi} \cdot 0 - \frac{1}{2}\right) - \left(-\frac{2}{\pi} \cdot 1 - 0\right)\right]$

$= 2\left(\frac{2}{\pi} - \frac{1}{2}\right) = 2\left(\frac{4-\pi}{2\pi}\right) = \frac{4-\pi}{\pi}$

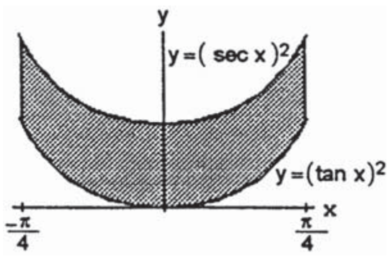


67. $a = -\frac{\pi}{4}, b = \frac{\pi}{4}$; $f(x) - g(x) = \sec^2 x - \tan^2 x$

$\Rightarrow A = \int_{-\pi/4}^{\pi/4} (\sec^2 x - \tan^2 x)\, dx$

$= \int_{-\pi/4}^{\pi/4} [\sec^2 x - (\sec^2 x - 1)]\, dx$

$= \int_{-\pi/4}^{\pi/4} 1 \cdot dx = [x]_{-\pi/4}^{\pi/4} = \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$

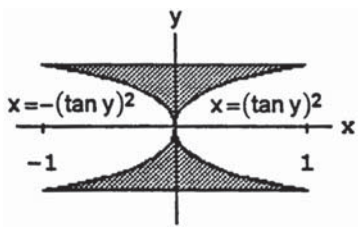


68. $c = -\frac{\pi}{4}, d = \frac{\pi}{4}$; $f(y) - g(y) = \tan^2 y - (-\tan^2 y) = 2 \tan^2 y$

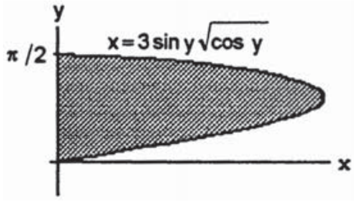
$= 2(\sec^2 y - 1) \Rightarrow A = \int_{-\pi/4}^{\pi/4} 2(\sec^2 y - 1)\, dy$

$= 2[\tan y - y]_{-\pi/4}^{\pi/4} = 2\left[\left(1 - \frac{\pi}{4}\right) - \left(-1 + \frac{\pi}{4}\right)\right]$

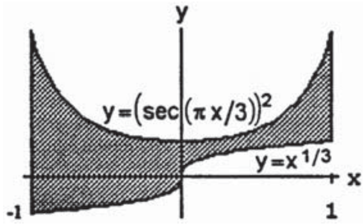
$= 4\left(1 - \frac{\pi}{4}\right) = 4 - \pi$



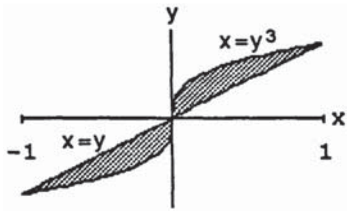
69. $c = 0, d = \frac{\pi}{2}$; $f(y) - g(y) = 3 \sin y \sqrt{\cos y} - 0 = 3 \sin y \sqrt{\cos y}$
$\Rightarrow A = 3\int_0^{\pi/2} \sin y \sqrt{\cos y}\, dy = -3\left[\frac{2}{3}(\cos y)^{3/2}\right]_0^{\pi/2}$
$= -2(0 - 1) = 2$



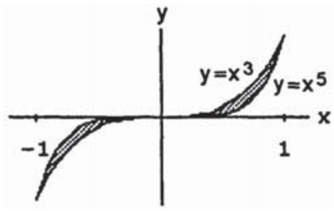
70. $a = -1, b = 1$; $f(x) - g(x) = \sec^2\left(\frac{\pi x}{3}\right) - x^{1/3}$
$\Rightarrow A = \int_{-1}^{1}\left[\sec^2\left(\frac{\pi x}{3}\right) - x^{1/3}\right] dx = \left[\frac{3}{\pi}\tan\left(\frac{\pi x}{3}\right) - \frac{3}{4}x^{4/3}\right]_{-1}^{1}$
$= \left(\frac{3}{\pi}\sqrt{3} - \frac{3}{4}\right) - \left[\frac{3}{\pi}\left(-\sqrt{3}\right) - \frac{3}{4}\right] = \frac{6\sqrt{3}}{\pi}$



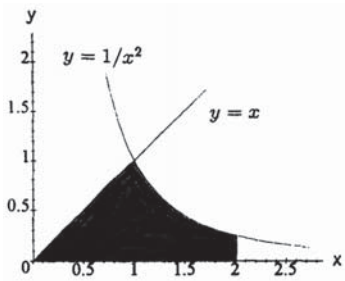
71. $A = A_1 + A_2$
Limits of integration: $x = y^3$ and $x = y \Rightarrow y = y^3$
$\Rightarrow y^3 - y = 0 \Rightarrow y(y-1)(y+1) = 0 \Rightarrow c_1 = -1, d_1 = 0$
and $c_2 = 0, d_2 = 1$; $f_1(y) - g_1(y) = y^3 - y$ and
$f_2(y) - g_2(y) = y - y^3 \Rightarrow$ by symmetry about the origin,
$A_1 + A_2 = 2A_2 \Rightarrow A = 2\int_0^1 (y - y^3)\, dy = 2\left[\frac{y^2}{2} - \frac{y^4}{4}\right]_0^1$
$= 2\left(\frac{1}{2} - \frac{1}{4}\right) = \frac{1}{2}$



72. $A = A_1 + A_2$
Limits of integration: $y = x^3$ and $y = x^5 \Rightarrow x^3 = x^5$
$\Rightarrow x^5 - x^3 = 0 \Rightarrow x^3(x-1)(x+1) = 0 \Rightarrow a_1 = -1, b_1 = 0$
and $a_2 = 0, b_2 = 1$; $f_1(x) - g_1(x) = x^3 - x^5$ and
$f_2(x) - g_2(x) = x^5 - x^3 \Rightarrow$ by symmetry about the origin,
$A_1 + A_2 = 2A_2 \Rightarrow A = 2\int_0^1 (x^3 - x^5)\, dx = 2\left[\frac{x^4}{4} - \frac{x^6}{6}\right]_0^1$
$= 2\left(\frac{1}{4} - \frac{1}{6}\right) = \frac{1}{6}$



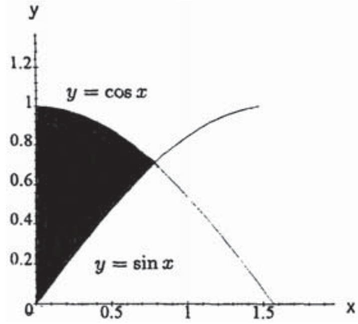
73. $A = A_1 + A_2$
Limits of integration: $y = x$ and $y = \frac{1}{x^2} \Rightarrow x = \frac{1}{x^2}, x \neq 0$
$\Rightarrow x^3 = 1 \Rightarrow x = 1$, $f_1(x) - g_1(x) = x - 0 = x$
$\Rightarrow A_1 = \int_0^1 x\, dx = \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2}$; $f_2(x) - g_2(x) = \frac{1}{x^2} - 0$
$= x^{-2} \Rightarrow A_2 = \int_1^2 x^{-2}\, dx = \left[\frac{-1}{x}\right]_1^2 = -\frac{1}{2} + 1 = \frac{1}{2}$;
$A = A_1 + A_2 = \frac{1}{2} + \frac{1}{2} = 1$



74. Limits of integration: $\sin x = \cos x \;\Rightarrow\; x = \frac{\pi}{4} \;\Rightarrow\; a = 0$ and $b = \frac{\pi}{4}$; $f(x) - g(x) = \cos x - \sin x$

$\Rightarrow\; A = \int_0^{\pi/4} (\cos x - \sin x)\,dx = [\sin x + \cos x]_0^{\pi/4}$

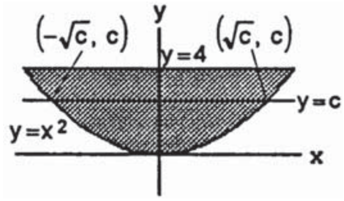
$= \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) - (0 + 1) = \sqrt{2} - 1$



75. (a) The coordinates of the points of intersection of the line and parabola are $c = x^2 \;\Rightarrow\; x = \pm\sqrt{c}$ and $y = c$

(b) $f(y) - g(y) = \sqrt{y} - \left(-\sqrt{y}\right) = 2\sqrt{y} \;\Rightarrow$ the area of the lower section is, $A_L = \int_0^c [f(y) - g(y)]\,dy$

$= 2\int_0^c \sqrt{y}\,dy = 2\left[\frac{2}{3}y^{3/2}\right]_0^c = \frac{4}{3}c^{3/2}$. The area of the entire shaded region can be found by setting $c = 4$: $A = \left(\frac{4}{3}\right)4^{3/2} = \frac{4\cdot 8}{3} = \frac{32}{3}$. Since we want c to divide the region into subsections of equal area we have $A = 2A_L \;\Rightarrow\; \frac{32}{3} = 2\left(\frac{4}{3}c^{3/2}\right) \;\Rightarrow\; c = 4^{2/3}$



(c) $f(x) - g(x) = c - x^2 \;\Rightarrow\; A_L = \int_{-\sqrt{c}}^{\sqrt{c}} [f(x) - g(x)]\,dx = \int_{-\sqrt{c}}^{\sqrt{c}} (c - x^2)\,dx = \left[cx - \frac{x^3}{3}\right]_{-\sqrt{c}}^{\sqrt{c}} = 2\left[c^{3/2} - \frac{c^{3/2}}{3}\right]$

$= \frac{4}{3}c^{3/2}$. Again, the area of the whole shaded region can be found by setting $c = 4 \;\Rightarrow\; A = \frac{32}{3}$. From the condition $A = 2A_L$, we get $\frac{4}{3}c^{3/2} = \frac{32}{3} \;\Rightarrow\; c = 4^{2/3}$ as in part (b).

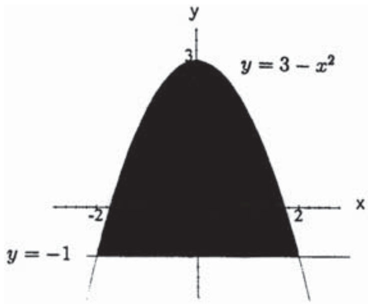
76. (a) Limits of integration: $y = 3 - x^2$ and $y = -1$

$\Rightarrow\; 3 - x^2 = -1 \;\Rightarrow\; x^2 = 4 \;\Rightarrow\; a = -2$ and $b = 2$;

$f(x) - g(x) = (3 - x^2) - (-1) = 4 - x^2$

$\Rightarrow\; A = \int_{-1}^{2} (4 - x^2)\,dx = \left[4x - \frac{x^3}{3}\right]_{-2}^{2}$

$= \left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right) = 16 - \frac{16}{3} = \frac{32}{3}$



(b) Limits of integration: let $x = 0$ in $y = 3 - x^2$

$\Rightarrow\; y = 3$; $f(y) - g(y) = \sqrt{3 - y} - \left(-\sqrt{3 - y}\right)$

$= 2(3 - y)^{1/2}$

$\Rightarrow\; A = 2\int_{-1}^{3} (3 - y)^{1/2}\,dy = -2\int_{-1}^{3} (3 - y)^{1/2}(-1)\,dy = (-2)\left[\frac{2(3 - y)^{3/2}}{3}\right]_{-1}^{3} = \left(-\frac{4}{3}\right)\left[0 - (3 + 1)^{3/2}\right]$

$= \left(\frac{4}{3}\right)(8) = \frac{32}{3}$

77. Limits of integration: $y = 1 + \sqrt{x}$ and $y = \frac{2}{\sqrt{x}}$

$\Rightarrow\; 1 + \sqrt{x} = \frac{2}{\sqrt{x}},\ x \neq 0 \;\Rightarrow\; \sqrt{x} + x = 2 \;\Rightarrow\; x = (2 - x)^2$

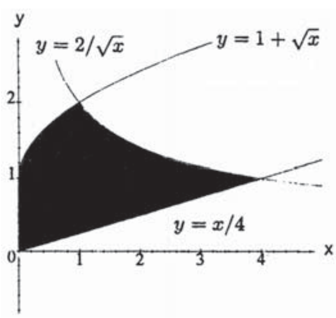
$\Rightarrow\; x = 4 - 4x + x^2 \;\Rightarrow\; x^2 - 5x + 4 = 0$

$\Rightarrow\; (x - 4)(x - 1) = 0 \;\Rightarrow\; x = 1, 4$ (but $x = 4$ does not satisfy the equation); $y = \frac{2}{\sqrt{x}}$ and $y = \frac{x}{4} \;\Rightarrow\; \frac{2}{\sqrt{x}} = \frac{x}{4}$

$\Rightarrow\; 8 = x\sqrt{x} \;\Rightarrow\; 64 = x^3 \;\Rightarrow\; x = 4$.

Therefore, AREA $= A_1 + A_2$: $f_1(x) - g_1(x) = \left(1 + x^{1/2}\right) - \frac{x}{4}$

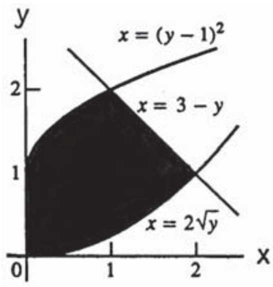
$\Rightarrow\; A_1 = \int_0^1 \left(1 + x^{1/2} - \frac{x}{4}\right)dx = \left[x + \frac{2}{3}x^{3/2} - \frac{x^2}{8}\right]_0^1$



$= \left(1 + \frac{2}{3} - \frac{1}{8}\right) - 0 = \frac{37}{24}$; $f_2(x) - g_2(x) = 2x^{-1/2} - \frac{x}{4} \;\Rightarrow\; A_2 = \int_1^4 \left(2x^{-1/2} - \frac{x}{4}\right)dx = \left[4x^{1/2} - \frac{x^2}{8}\right]_1^4$

$= \left(4\cdot 2 - \frac{16}{8}\right) - \left(4 - \frac{1}{8}\right) = 4 - \frac{15}{8} = \frac{17}{8}$; Therefore, AREA $= A_1 + A_2 = \frac{37}{24} + \frac{17}{8} = \frac{37+51}{24} = \frac{88}{24} = \frac{11}{3}$

78. Limits of integration: $(y-1)^2 = 3-y \Rightarrow y^2 - 2y + 1 = 3 - y \Rightarrow y^2 - y - 2 = 0 \Rightarrow (y-2)(y+1) = 0 \Rightarrow y = 2$ since $y > 0$; also, $2\sqrt{y} = 3 - y \Rightarrow 4y = 9 - 6y + y^2 \Rightarrow y^2 - 10y + 9 = 0 \Rightarrow (y-9)(y-1) = 0 \Rightarrow y = 1$ since $y = 9$ does not satisfy the equation;



AREA $= A_1 + A_2$

$f_1(y) - g_1(y) = 2\sqrt{y} - 0 = 2y^{1/2}$

$\Rightarrow A_1 = 2\int_0^1 y^{1/2}\,dy = 2\left[\frac{2y^{3/2}}{3}\right]_0^1 = \frac{4}{3}$; $f_2(y) - g_2(y) = (3-y) - (y-1)^2$

$\Rightarrow A_2 = \int_1^2 [3 - y - (y-1)^2]\,dy = \left[3y - \frac{1}{2}y^2 - \frac{1}{3}(y-1)^3\right]_1^2 = \left(6 - 2 - \frac{1}{3}\right) - \left(3 - \frac{1}{2} + 0\right) = 1 - \frac{1}{3} + \frac{1}{2} = \frac{7}{6}$;

Therefore, $A_1 + A_2 = \frac{4}{3} + \frac{7}{6} = \frac{15}{6} = \frac{5}{2}$

79. Area between parabola and $y = a^2$: $A = 2\int_0^a (a^2 - x^2)\,dx = 2\left[a^2x - \frac{1}{3}x^3\right]_0^a = 2\left(a^3 - \frac{a^3}{3}\right) - 0 = \frac{4a^3}{3}$;

Area of triangle AOC: $\frac{1}{2}(2a)(a^2) = a^3$; limit of ratio $= \lim_{a \to 0^+} \frac{a^3}{\left(\frac{4a^3}{3}\right)} = \frac{3}{4}$ which is independent of a.

80. $A = \int_a^b 2f(x)\,dx - \int_a^b f(x)\,dx = 2\int_a^b f(x)\,dx - \int_a^b f(x)\,dx = \int_a^b f(x)\,dx = 4$

81. Neither one; they are both zero. Neither integral takes into account the changes in the formulas for the region's upper and lower bounding curves at $x = 0$. The area of the shaded region is actually
$A = \int_{-1}^0 [-x - (x)]\,dx + \int_0^1 [x - (-x)]\,dx = \int_{-1}^0 -2x\,dx + \int_0^1 2x\,dx = 2.$

82. It is sometimes true. It is true if $f(x) \ge g(x)$ for all x between a and b. Otherwise it is false. If the graph of f lies below the graph of g for a portion of the interval of integration, the integral over that portion will be negative and the integral over $[a, b]$ will be less than the area between the curves (see Exercise 53).

83. Let $u = 2x \Rightarrow du = 2\,dx \Rightarrow \frac{1}{2}\,du = dx$; $x = 1 \Rightarrow u = 2$, $x = 3 \Rightarrow u = 6$
$\int_1^3 \frac{\sin 2x}{x}\,dx = \int_2^6 \frac{\sin u}{\left(\frac{u}{2}\right)}\left(\frac{1}{2}\,du\right) = \int_2^6 \frac{\sin u}{u}\,du = [F(u)]_2^6 = F(6) - F(2)$

84. Let $u = 1 - x \Rightarrow du = -dx \Rightarrow -du = dx$; $x = 0 \Rightarrow u = 1$, $x = 1 \Rightarrow u = 0$
$\int_0^1 f(1-x)\,dx = \int_1^0 f(u)(-du) = -\int_1^0 f(u)\,du = \int_0^1 f(u)\,du = \int_0^1 f(x)\,dx$

85. (a) Let $u = -x \Rightarrow du = -dx$; $x = -1 \Rightarrow u = 1$, $x = 0 \Rightarrow u = 0$
f odd $\Rightarrow f(-x) = -f(x)$. Then $\int_{-1}^0 f(x)\,dx = \int_1^0 f(-u)(-du) = \int_1^0 -f(u)(-du) = \int_1^0 f(u)\,du = -\int_0^1 f(u)\,du = -3$

(b) Let $u = -x \Rightarrow du = -dx$; $x = -1 \Rightarrow u = 1$, $x = 0 \Rightarrow u = 0$
f even $\Rightarrow f(-x) = f(x)$. Then $\int_{-1}^0 f(x)\,dx = \int_1^0 f(-u)(-du) = -\int_1^0 f(u)\,du = \int_0^1 f(u)\,du = 3$

86. (a) Consider $\int_{-a}^0 f(x)\,dx$ when f is odd. Let $u = -x \Rightarrow du = -dx \Rightarrow -du = dx$ and $x = -a \Rightarrow u = a$ and $x = 0 \Rightarrow u = 0$. Thus $\int_{-a}^0 f(x)\,dx = \int_a^0 -f(-u)\,du = \int_a^0 f(u)\,du = -\int_0^a f(u)\,du = -\int_0^a f(x)\,dx.$
Thus $\int_{-a}^a f(x)\,dx = \int_{-a}^0 f(x)\,dx + \int_0^a f(x)\,dx = -\int_0^a f(x)\,dx + \int_0^a f(x)\,dx = 0.$