54. $R(x) = \frac{rx}{h} \Rightarrow V = \int_0^h \pi[R(x)]^2\,dx = \pi\int_0^h \frac{r^2x^2}{h^2}\,dx = \frac{\pi r^2}{h^2}\left[\frac{x^3}{3}\right]_0^h = \left(\frac{\pi r^2}{h^2}\right)\left(\frac{h^3}{3}\right) = \frac{1}{3}\pi r^2 h$, the volume of a cone of radius r and height h.

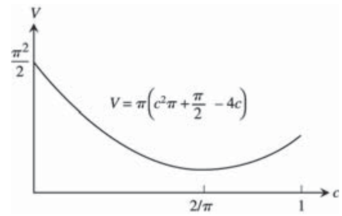
55. $R(y) = \sqrt{256 - y^2} \Rightarrow V = \int_{-16}^{-7} \pi[R(y)]^2\,dy = \pi\int_{-16}^{-7}(256 - y^2)\,dy = \pi\left[256y - \frac{y^3}{3}\right]_{-16}^{-7}$
$= \pi\left[(256)(-7) + \frac{7^3}{3} - \left((256)(-16) + \frac{16^3}{3}\right)\right] = \pi\left(\frac{7^3}{3} + 256(16-7) - \frac{16^3}{3}\right) = 1053\pi\ \text{cm}^3 \approx 3308\ \text{cm}^3$

56. $R(x) = \frac{x}{12}\sqrt{36 - x^2} \Rightarrow V = \int_0^6 \pi[R(x)]^2\,dx = \pi\int_0^6 \frac{x^2}{144}(36 - x^2)\,dx = \frac{\pi}{144}\int_0^6 (36x^2 - x^4)\,dx$
$= \frac{\pi}{144}\left[12x^3 - \frac{x^5}{5}\right]_0^6 = \frac{\pi}{144}\left(12\cdot 6^3 - \frac{6^5}{5}\right) = \frac{\pi\cdot 6^3}{144}\left(12 - \frac{36}{5}\right) = \left(\frac{196\pi}{144}\right)\left(\frac{60-36}{5}\right) = \frac{36\pi}{5}\ \text{cm}^3$. The plumb bob will weigh about $W = (8.5)\left(\frac{36\pi}{5}\right) \approx 192$ gm, to the nearest gram.

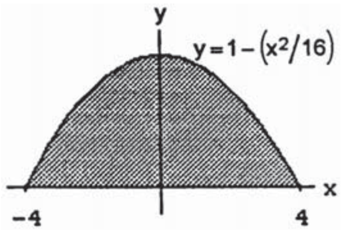
57. (a) $R(x) = |c - \sin x|$, so $V = \pi\int_0^\pi [R(x)]^2\,dx = \pi\int_0^\pi (c - \sin x)^2\,dx = \pi\int_0^\pi (c^2 - 2c\sin x + \sin^2 x)\,dx$
$= \pi\int_0^\pi \left(c^2 - 2c\sin x + \frac{1-\cos 2x}{2}\right)dx = \pi\int_0^\pi \left(c^2 + \frac{1}{2} - 2c\sin x - \frac{\cos 2x}{2}\right)dx$
$= \pi\left[\left(c^2 + \frac{1}{2}\right)x + 2c\cos x - \frac{\sin 2x}{4}\right]_0^\pi = \pi\left[\left(c^2\pi + \frac{\pi}{2} - 2c - 0\right) - (0 + 2c - 0)\right] = \pi\left(c^2\pi + \frac{\pi}{2} - 4c\right)$. Let $V(c) = \pi\left(c^2\pi + \frac{\pi}{2} - 4c\right)$. We find the extreme values of V(c): $\frac{dV}{dc} = \pi(2c\pi - 4) = 0 \Rightarrow c = \frac{2}{\pi}$ is a critical point, and $V\left(\frac{2}{\pi}\right) = \pi\left(\frac{4}{\pi} + \frac{\pi}{2} - \frac{8}{\pi}\right) = \pi\left(\frac{\pi}{2} - \frac{4}{\pi}\right) = \frac{\pi^2}{2} - 4$; Evaluate V at the endpoints: $V(0) = \frac{\pi^2}{2}$ and $V(1) = \pi\left(\frac{3}{2}\pi - 4\right) = \frac{\pi^2}{2} - (4 - \pi)\pi$. Now we see that the function's absolute minimum value is $\frac{\pi^2}{2} - 4$, taken on at the critical point $c = \frac{2}{\pi}$. (See also the accompanying graph.)

(b) From the discussion in part (a) we conclude that the function's absolute maximum value is $\frac{\pi^2}{2}$, taken on at the endpoint $c = 0$.

(c) The graph of the solid's volume as a function of c for $0 \le c \le 1$ is given at the right. As c moves away from $[0, 1]$ the volume of the solid increases without bound. If we approximate the solid as a set of solid disks, we can see that the radius of a typical disk increases without bounds as c moves away from [0, 1].



58. (a) $R(x) = 1 - \frac{x^2}{16} \Rightarrow V = \int_{-4}^{4} \pi[R(x)]^2\,dx$
$= \pi\int_{-4}^{4}\left(1 - \frac{x^2}{16}\right)^2 dx = \pi\int_{-4}^{4}\left(1 - \frac{x^2}{8} + \frac{x^4}{16^2}\right)dx$
$= \pi\left[x - \frac{x^3}{24} + \frac{x^5}{5\cdot 16^2}\right]_{-4}^{4} = 2\pi\left(4 - \frac{4^3}{24} + \frac{4^5}{5\cdot 16^2}\right)$
$= 2\pi\left(4 - \frac{8}{3} + \frac{4}{5}\right) = \frac{2\pi}{15}(60 - 40 + 12) = \frac{64\pi}{15}\ \text{ft}^3$



(b) The helicopter will be able to fly $\left(\frac{64\pi}{15}\right)(7.481)(2) \approx 201$ additional miles.

6.2 VOLUME BY CYLINDRICAL SHELLS

1. For the sketch given, a = 0, b = 2;
$V = \int_a^b 2\pi\left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right)dx = \int_0^2 2\pi x\left(1 + \frac{x^2}{4}\right)dx = 2\pi\int_0^2\left(x + \frac{x^3}{4}\right)dx = 2\pi\left[\frac{x^2}{2} + \frac{x^4}{16}\right]_0^2 = 2\pi\left(\frac{4}{2} + \frac{16}{16}\right)$
$= 2\pi\cdot 3 = 6\pi$

2. For the sketch given, a = 0, b = 2;
$V = \int_a^b 2\pi\left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right)dx = \int_0^2 2\pi x\left(2 - \frac{x^2}{4}\right)dx = 2\pi\int_0^2\left(2x - \frac{x^3}{4}\right)dx = 2\pi\left[x^2 - \frac{x^4}{16}\right]_0^2 = 2\pi(4 - 1) = 6\pi$

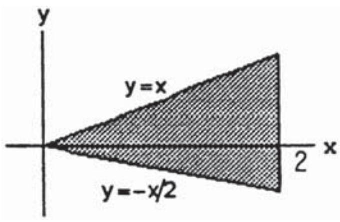
3. For the sketch given, $c = 0$, $d = \sqrt{2}$;
$V = \int_c^d 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dy = \int_0^{\sqrt{2}} 2\pi y \cdot (y^2)\, dy = 2\pi \int_0^{\sqrt{2}} y^3\, dy = 2\pi \left[\frac{y^4}{4}\right]_0^{\sqrt{2}} = 2\pi$

4. For the sketch given, $c = 0$, $d = \sqrt{3}$;
$V = \int_c^d 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dy = \int_0^{\sqrt{3}} 2\pi y \cdot [3 - (3 - y^2)]\, dy = 2\pi \int_0^{\sqrt{3}} y^3\, dy = 2\pi \left[\frac{y^4}{4}\right]_0^{\sqrt{3}} = \frac{9\pi}{2}$

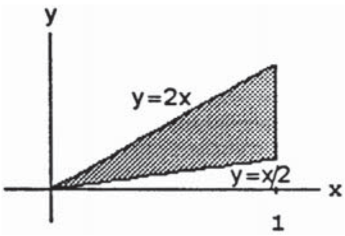
5. For the sketch given, $a = 0$, $b = \sqrt{3}$;
$V = \int_a^b 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dx = \int_0^{\sqrt{3}} 2\pi x \cdot \left(\sqrt{x^2+1}\right) dx$;
$\left[u = x^2 + 1 \Rightarrow du = 2x\, dx;\ x = 0 \Rightarrow u = 1,\ x = \sqrt{3} \Rightarrow u = 4\right]$
$\to V = \pi \int_1^4 u^{1/2}\, du = \pi \left[\frac{2}{3} u^{3/2}\right]_1^4 = \frac{2\pi}{3}\left(4^{3/2} - 1\right) = \left(\frac{2\pi}{3}\right)(8 - 1) = \frac{14\pi}{3}$

6. For the sketch given, $a = 0$, $b = 3$;
$V = \int_a^b 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dx = \int_0^3 2\pi x \left(\frac{9x}{\sqrt{x^3+9}}\right) dx$;
$[u = x^3 + 9 \Rightarrow du = 3x^2\, dx \Rightarrow 3\, du = 9x^2\, dx;\ x = 0 \Rightarrow u = 9,\ x = 3 \Rightarrow u = 36]$
$\to V = 2\pi \int_9^{36} 3u^{-1/2}\, du = 6\pi \left[2u^{1/2}\right]_9^{36} = 12\pi\left(\sqrt{36} - \sqrt{9}\right) = 36\pi$

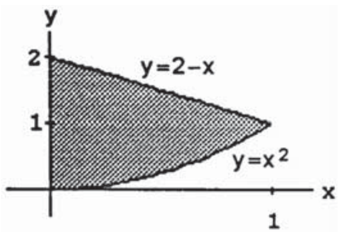
7. $a = 0$, $b = 2$;
$V = \int_a^b 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dx = \int_0^2 2\pi x \left[x - \left(-\frac{x}{2}\right)\right] dx$
$= \int_0^2 2\pi x^2 \cdot \frac{3}{2}\, dx = \pi \int_0^2 3x^2\, dx = \pi \left[x^3\right]_0^2 = 8\pi$



8. $a = 0$, $b = 1$;
$V = \int_a^b 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dx = \int_0^1 2\pi x \left(2x - \frac{x}{2}\right) dx$
$= \pi \int_0^1 2\left(\frac{3x^2}{2}\right) dx = \pi \int_0^1 3x^2\, dx = \pi \left[x^3\right]_0^1 = \pi$

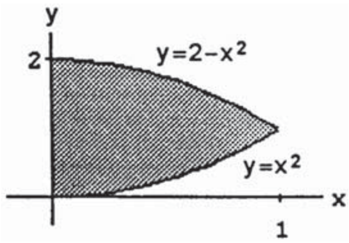


9. $a = 0$, $b = 1$;
$V = \int_a^b 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dx = \int_0^1 2\pi x \left[(2 - x) - x^2\right] dx$
$= 2\pi \int_0^1 (2x - x^2 - x^3)\, dx = 2\pi \left[x^2 - \frac{x^3}{3} - \frac{x^4}{4}\right]_0^1$
$= 2\pi\left(1 - \frac{1}{3} - \frac{1}{4}\right) = 2\pi\left(\frac{12 - 4 - 3}{12}\right) = \frac{10\pi}{12} = \frac{5\pi}{6}$



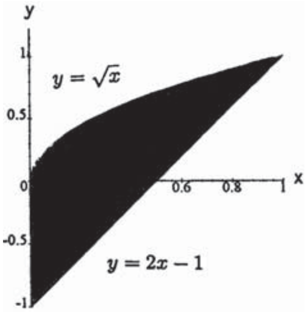
10. $a = 0, b = 1;$

$$V = \int_a^b 2\pi \left(\substack{\text{shell}\\ \text{radius}}\right)\left(\substack{\text{shell}\\ \text{height}}\right) dx = \int_0^1 2\pi x\left[(2 - x^2) - x^2\right] dx$$
$$= 2\pi \int_0^1 x\left(2 - 2x^2\right) dx = 4\pi \int_0^1 \left(x - x^3\right) dx$$
$$= 4\pi \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = 4\pi\left(\frac{1}{2} - \frac{1}{4}\right) = \pi$$



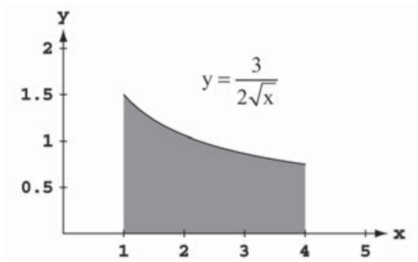
11. $a = 0, b = 1;$

$$V = \int_a^b 2\pi \left(\substack{\text{shell}\\ \text{radius}}\right)\left(\substack{\text{shell}\\ \text{height}}\right) dx = \int_0^1 2\pi x\left[\sqrt{x} - (2x - 1)\right] dx$$
$$= 2\pi \int_0^1 \left(x^{3/2} - 2x^2 + x\right) dx = 2\pi\left[\frac{2}{5}x^{5/2} - \frac{2}{3}x^3 + \frac{1}{2}x^2\right]_0^1$$
$$= 2\pi\left(\frac{2}{5} - \frac{2}{3} + \frac{1}{2}\right) = 2\pi\left(\frac{12 - 20 + 15}{30}\right) = \frac{7\pi}{15}$$



12. $a = 1, b = 4;$

$$V = \int_a^b 2\pi \left(\substack{\text{shell}\\ \text{radius}}\right)\left(\substack{\text{shell}\\ \text{height}}\right) dx = \int_1^4 2\pi x\left(\frac{3}{2}x^{-1/2}\right) dx$$
$$= 3\pi \int_1^4 x^{1/2}\, dx = 3\pi\left[\frac{2}{3}x^{3/2}\right]_1^4 = 2\pi\left(4^{3/2} - 1\right)$$
$$= 2\pi(8 - 1) = 14\pi$$



13. (a) $xf(x) = \begin{cases} x \cdot \frac{\sin x}{x}, & 0 < x \le \pi \\ x, & x = 0 \end{cases} \Rightarrow xf(x) = \begin{cases} \sin x, & 0 < x \le \pi \\ 0, & x = 0 \end{cases}$; since $\sin 0 = 0$ we have

$xf(x) = \begin{cases} \sin x, & 0 < x \le \pi \\ \sin x, & x = 0 \end{cases} \Rightarrow xf(x) = \sin x, 0 \le x \le \pi$

(b) $V = \int_a^b 2\pi \left(\substack{\text{shell}\\ \text{radius}}\right)\left(\substack{\text{shell}\\ \text{height}}\right) dx = \int_0^\pi 2\pi x \cdot f(x)\, dx$ and $x \cdot f(x) = \sin x, 0 \le x \le \pi$ by part (a)

$\Rightarrow V = 2\pi \int_0^\pi \sin x\, dx = 2\pi[-\cos x]_0^\pi = 2\pi(-\cos \pi + \cos 0) = 4\pi$

14. (a) $xg(x) = \begin{cases} x \cdot \frac{\tan^2 x}{x}, & 0 < x \le \frac{\pi}{4} \\ x \cdot 0, & x = 0 \end{cases} \Rightarrow xg(x) = \begin{cases} \tan^2 x, & 0 < x \le \pi/4 \\ 0, & x = 0 \end{cases}$; since $\tan 0 = 0$ we have

$xg(x) = \begin{cases} \tan^2 x, & 0 < x \le \pi/4 \\ \tan^2 x, & x = 0 \end{cases} \Rightarrow xg(x) = \tan^2 x, 0 \le x \le \pi/4$

(b) $V = \int_a^b 2\pi \left(\substack{\text{shell}\\ \text{radius}}\right)\left(\substack{\text{shell}\\ \text{height}}\right) dx = \int_0^{\pi/4} 2\pi x \cdot g(x)\, dx$ and $x \cdot g(x) = \tan^2 x, 0 \le x \le \pi/4$ by part (a)

$\Rightarrow V = 2\pi \int_0^{\pi/4} \tan^2 x\, dx = 2\pi \int_0^{\pi/4} (\sec^2 x - 1)\, dx = 2\pi[\tan x - x]_0^{\pi/4} = 2\pi\left(1 - \frac{\pi}{4}\right) = \frac{4\pi - \pi^2}{2}$

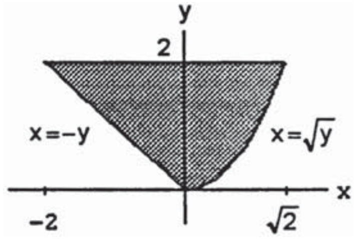
15. $c = 0, d = 2;$

$V = \int_c^d 2\pi \left(\frac{\text{shell}}{\text{radius}}\right)\left(\frac{\text{shell}}{\text{height}}\right) dy = \int_0^2 2\pi y\left[\sqrt{y} - (-y)\right] dy$

$= 2\pi \int_0^2 \left(y^{3/2} + y^2\right) dy = 2\pi \left[\frac{2y^{5/2}}{5} + \frac{y^3}{3}\right]_0^2$

$= 2\pi \left[\frac{2}{5}\left(\sqrt{2}\right)^5 + \frac{2^3}{3}\right] = 2\pi\left(\frac{8\sqrt{2}}{5} + \frac{8}{3}\right) = 16\pi\left(\frac{\sqrt{2}}{5} + \frac{1}{3}\right)$

$= \frac{16\pi}{15}\left(3\sqrt{2} + 5\right)$

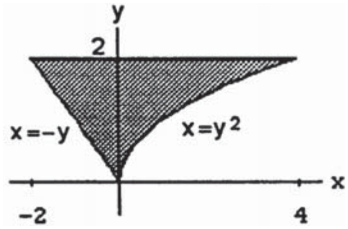


16. $c = 0, d = 2;$

$V = \int_c^d 2\pi \left(\frac{\text{shell}}{\text{radius}}\right)\left(\frac{\text{shell}}{\text{height}}\right) dy = \int_0^2 2\pi y\,[y^2 - (-y)]dy$

$= 2\pi \int_0^2 \left(y^3 + y^2\right) dy = 2\pi \left[\frac{y^4}{4} + \frac{y^3}{3}\right]_0^2 = 16\pi\left(\frac{2}{4} + \frac{1}{3}\right)$

$= 16\pi\left(\frac{5}{6}\right) = \frac{40\pi}{3}$

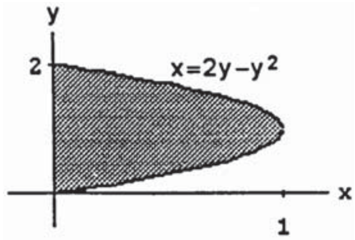


17. $c = 0, d = 2;$

$V = \int_c^d 2\pi \left(\frac{\text{shell}}{\text{radius}}\right)\left(\frac{\text{shell}}{\text{height}}\right) dy = \int_0^2 2\pi y\,(2y - y^2)dy$

$= 2\pi \int_0^2 \left(2y^2 - y^3\right) dy = 2\pi \left[\frac{2y^3}{3} - \frac{y^4}{4}\right]_0^2 = 2\pi\left(\frac{16}{3} - \frac{16}{4}\right)$

$= 32\pi\left(\frac{1}{3} - \frac{1}{4}\right) = \frac{32\pi}{12} = \frac{8\pi}{3}$

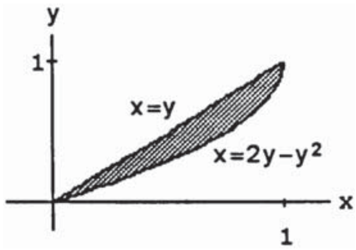


18. $c = 0, d = 1;$

$V = \int_c^d 2\pi \left(\frac{\text{shell}}{\text{radius}}\right)\left(\frac{\text{shell}}{\text{height}}\right) dy = \int_0^1 2\pi y\,(2y - y^2 - y)dy$

$= 2\pi \int_0^1 y\left(y - y^2\right) dy = 2\pi \int_0^1 \left(y^2 - y^3\right) dy$

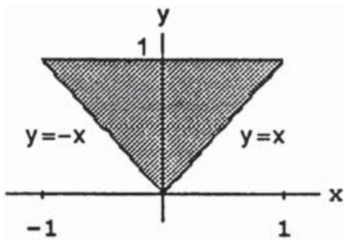
$= 2\pi \left[\frac{y^3}{3} - \frac{y^4}{4}\right]_0^1 = 2\pi\left(\frac{1}{3} - \frac{1}{4}\right) = \frac{\pi}{6}$



19. $c = 0, d = 1;$

$V = \int_c^d 2\pi \left(\frac{\text{shell}}{\text{radius}}\right)\left(\frac{\text{shell}}{\text{height}}\right) dy = 2\pi \int_0^1 y[y - (-y)]dy$

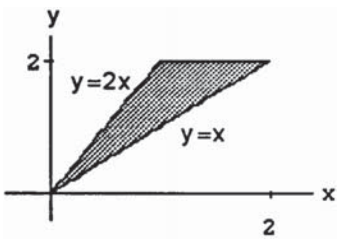
$= 2\pi \int_0^1 2y^2\, dy = \frac{4\pi}{3}\left[y^3\right]_0^1 = \frac{4\pi}{3}$



20. $c = 0, d = 2;$

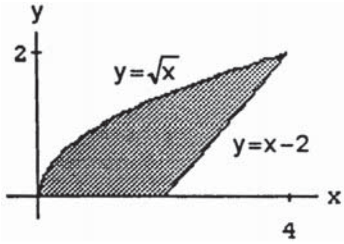
$V = \int_c^d 2\pi \left(\frac{\text{shell}}{\text{radius}}\right)\left(\frac{\text{shell}}{\text{height}}\right) dy = \int_0^2 2\pi\, y\left(y - \frac{y}{2}\right)dy$

$= 2\pi \int_0^2 \frac{y^2}{2} dy = \frac{\pi}{3}\left[y^3\right]_0^1 = \frac{8\pi}{3}$



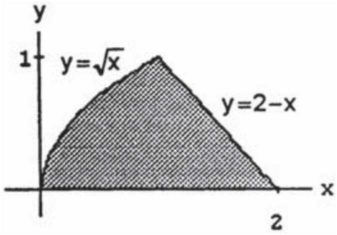
21. $c = 0, d = 2;$

$$V = \int_c^d 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dy = \int_0^2 2\pi y\left[(2+y) - y^2\right] dy$$
$$= 2\pi \int_0^2 \left(2y + y^2 - y^3\right) dy = 2\pi \left[y^2 + \frac{y^3}{3} - \frac{y^4}{4}\right]_0^2$$
$$= 2\pi\left(4 + \frac{8}{3} - \frac{16}{4}\right) = \frac{\pi}{6}(48 + 32 - 48) = \frac{16\pi}{3}$$



22. $c = 0, d = 1;$

$$V = \int_c^d 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dy = \int_0^1 2\pi y\left[(2-y) - y^2\right] dy$$
$$= 2\pi \int_0^1 \left(2y - y^2 - y^3\right) dy = 2\pi \left[y^2 - \frac{y^3}{3} - \frac{y^4}{4}\right]_0^1$$
$$= 2\pi\left(1 - \frac{1}{3} - \frac{1}{4}\right) = \frac{\pi}{6}(12 - 4 - 3) = \frac{5\pi}{6}$$



23. (a) $V = \int_c^d 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dy = \int_0^1 2\pi y \cdot 12\left(y^2 - y^3\right) dy = 24\pi \int_0^1 \left(y^3 - y^4\right) dy = 24\pi\left[\frac{y^4}{4} - \frac{y^5}{5}\right]_0^1$
$= 24\pi\left(\frac{1}{4} - \frac{1}{5}\right) = \frac{24\pi}{20} = \frac{6\pi}{5}$

(b) $V = \int_c^d 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dy = \int_0^1 2\pi(1-y)\left[12\left(y^2 - y^3\right)\right] dy = 24\pi \int_0^1 (1-y)\left(y^2 - y^3\right) dy$
$= 24\pi \int_0^1 \left(y^2 - 2y^3 + y^4\right) dy = 24\pi\left[\frac{y^3}{3} - \frac{y^4}{2} + \frac{y^5}{5}\right]_0^1 = 24\pi\left(\frac{1}{3} - \frac{1}{2} + \frac{1}{5}\right) = 24\pi\left(\frac{1}{30}\right) = \frac{4\pi}{5}$

(c) $V = \int_c^d 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dy = \int_0^1 2\pi\left(\frac{8}{5} - y\right)\left[12\left(y^2 - y^3\right)\right] dy = 24\pi \int_0^1 \left(\frac{8}{5} - y\right)\left(y^2 - y^3\right) dy$
$= 24\pi \int_0^1 \left(\frac{8}{5}y^2 - \frac{13}{5}y^3 + y^4\right) dy = 24\pi\left[\frac{8}{15}y^3 - \frac{13}{20}y^4 + \frac{y^5}{5}\right]_0^1 = 24\pi\left(\frac{8}{15} - \frac{13}{20} + \frac{1}{5}\right) = \frac{24\pi}{60}(32 - 39 + 12)$
$= \frac{24\pi}{12} = 2\pi$

(d) $V = \int_c^d 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dy = \int_0^1 2\pi\left(y + \frac{2}{5}\right)\left[12\left(y^2 - y^3\right)\right] dy = 24\pi \int_0^1 \left(y + \frac{2}{5}\right)\left(y^2 - y^3\right) dy$
$= 24\pi \int_0^1 \left(y^3 - y^4 + \frac{2}{5}y^2 - \frac{2}{5}y^3\right) dy = 24\pi \int_0^1 \left(\frac{2}{5}y^2 + \frac{3}{5}y^3 - y^4\right) dy = 24\pi\left[\frac{2}{15}y^3 + \frac{3}{20}y^4 - \frac{y^5}{5}\right]_0^1$
$= 24\pi\left(\frac{2}{15} + \frac{3}{20} - \frac{1}{5}\right) = \frac{24\pi}{60}(8 + 9 - 12) = \frac{24\pi}{12} = 2\pi$

24. (a) $V = \int_c^d 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dy = \int_0^2 2\pi y\left[\frac{y^2}{2} - \left(\frac{y^4}{4} - \frac{y^2}{2}\right)\right] dy = \int_0^2 2\pi y\left(y^2 - \frac{y^4}{4}\right) dy = 2\pi \int_0^2 \left(y^3 - \frac{y^5}{4}\right) dy$
$= 2\pi\left[\frac{y^4}{4} - \frac{y^6}{24}\right]_0^2 = 2\pi\left(\frac{2^4}{4} - \frac{2^6}{24}\right) = 32\pi\left(\frac{1}{4} - \frac{4}{24}\right) = 32\pi\left(\frac{1}{4} - \frac{1}{6}\right) = 32\pi\left(\frac{2}{24}\right) = \frac{8\pi}{3}$

(b) $V = \int_c^d 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dy = \int_0^2 2\pi(2-y)\left[\frac{y^2}{2} - \left(\frac{y^4}{4} - \frac{y^2}{2}\right)\right] dy = \int_0^2 2\pi(2-y)\left(y^2 - \frac{y^4}{4}\right) dy$
$= 2\pi \int_0^2 \left(2y^2 - \frac{y^4}{2} - y^3 + \frac{y^5}{4}\right) dy = 2\pi\left[\frac{2y^3}{3} - \frac{y^5}{10} - \frac{y^4}{4} + \frac{y^6}{24}\right]_0^2 = 2\pi\left(\frac{16}{3} - \frac{32}{10} - \frac{16}{4} + \frac{64}{24}\right) = \frac{8\pi}{5}$

(c) $V = \int_c^d 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dy = \int_0^2 2\pi(5-y)\left[\frac{y^2}{2} - \left(\frac{y^4}{4} - \frac{y^2}{2}\right)\right] dy = \int_0^2 2\pi(5-y)\left(y^2 - \frac{y^4}{4}\right) dy$
$= 2\pi \int_0^2 \left(5y^2 - \frac{5}{4}y^4 - y^3 + \frac{y^5}{4}\right) dy = 2\pi\left[\frac{5y^3}{3} - \frac{5y^5}{20} - \frac{y^4}{4} + \frac{y^6}{24}\right]_0^2 = 2\pi\left(\frac{40}{3} - \frac{160}{20} - \frac{16}{4} + \frac{64}{24}\right) = 8\pi$

(d) $V = \int_c^d 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dy = \int_0^2 2\pi\left(y + \frac{5}{8}\right)\left[\frac{y^2}{2} - \left(\frac{y^4}{4} - \frac{y^2}{2}\right)\right] dy = \int_0^2 2\pi\left(y + \frac{5}{8}\right)\left(y^2 - \frac{y^4}{4}\right) dy$
$= 2\pi \int_0^2 \left(y^3 - \frac{y^5}{4} + \frac{5}{8}y^2 - \frac{5}{32}y^4\right) dy = 2\pi\left[\frac{y^4}{4} - \frac{y^6}{24} + \frac{5y^3}{24} - \frac{5y^5}{160}\right]_0^2 = 2\pi\left(\frac{16}{4} - \frac{64}{24} + \frac{40}{24} - \frac{160}{160}\right) = 4\pi$

25. (a) About x-axis: $V = \int_c^d 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right)dy$

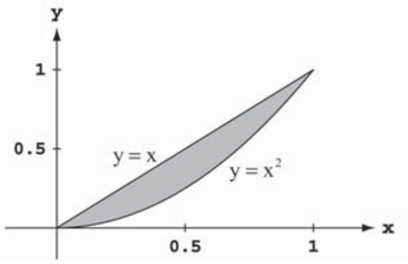
$= \int_0^1 2\pi y\left(\sqrt{y} - y\right)dy = 2\pi\int_0^1 \left(y^{3/2} - y^2\right)dy$

$= 2\pi\left[\frac{2}{5}y^{5/2} - \frac{1}{3}y^3\right]_0^1 = 2\pi\left(\frac{2}{5} - \frac{1}{3}\right) = \frac{2\pi}{15}$

About y-axis: $V = \int_a^b 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right)dx$

$= \int_0^1 2\pi x(x - x^2)dx = 2\pi\int_0^1 (x^2 - x^3)dx$

$= 2\pi\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = 2\pi\left(\frac{1}{3} - \frac{1}{4}\right) = \frac{\pi}{6}$



(b) About x-axis: $R(x) = x$ and $r(x) = x^2 \Rightarrow V = \int_a^b \pi\left[R(x)^2 - r(x)^2\right]dx = \int_0^1 \pi[x^2 - x^4]dx$

$= \pi\left[\frac{x^3}{3} - \frac{x^5}{5}\right]_0^1 = \pi\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{2\pi}{15}$

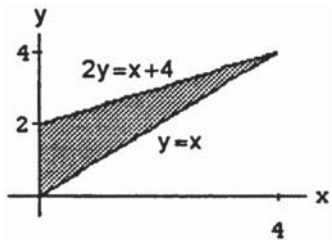
About y-axis: $R(y) = \sqrt{y}$ and $r(y) = y \Rightarrow V = \int_c^d \pi\left[R(y)^2 - r(y)^2\right]dy = \int_0^1 \pi[y - y^2]dy$

$= \pi\left[\frac{y^2}{2} - \frac{y^3}{3}\right]_0^1 = \pi\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{\pi}{6}$

26. (a) $V = \int_a^b \pi\left[R(x)^2 - r(x)^2\right]dx = \pi\int_0^4 \left[\left(\frac{x}{2} + 2\right)^2 - x^2\right]dx$

$= \pi\int_0^4 \left(-\frac{3}{4}x^2 + 2x + 4\right)dx = \pi\left[-\frac{x^3}{4} + x^2 + 4x\right]_0^4$

$= \pi(-16 + 16 + 16) = 16\pi$



(b) $V = \int_a^b 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right)dx = \int_0^4 2\pi x\left(\frac{x}{2} + 2 - x\right)dx$

$= \int_0^4 2\pi x\left(2 - \frac{x}{2}\right)dx = 2\pi\int_0^4 \left(2x - \frac{x^2}{2}\right)dx$

$= 2\pi\left[x^2 - \frac{x^3}{6}\right]_0^4 = 2\pi\left(16 - \frac{64}{6}\right) = \frac{32\pi}{3}$

(c) $V = \int_a^b 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right)dx = \int_0^4 2\pi(4 - x)\left(\frac{x}{2} + 2 - x\right)dx = \int_0^4 2\pi(4 - x)\left(2 - \frac{x}{2}\right)dx = 2\pi\int_0^4 \left(8 - 4x + \frac{x^2}{2}\right)dx$

$= 2\pi\left[8x - 2x^2 + \frac{x^3}{6}\right]_0^4 = 2\pi\left(32 - 32 + \frac{64}{6}\right) = \frac{64\pi}{3}$

(d) $V = \int_a^b \pi\left[R(x)^2 - r(x)^2\right]dx = \pi\int_0^4 \left[(8 - x)^2 - \left(6 - \frac{x}{2}\right)^2\right]dx = \pi\int_0^4 \left[(64 - 16x + x^2) - \left(36 - 6x + \frac{x^2}{4}\right)\right]dx$

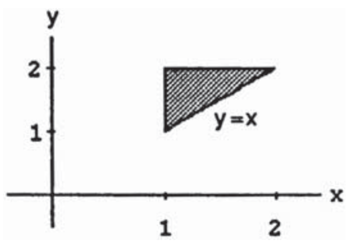
$\pi\int_0^4 \left(\frac{3}{4}x^2 - 10x + 28\right)dx = \pi\left[\frac{x^3}{4} - 5x^2 + 28x\right]_0^4 = \pi\left[16 - (5)(16) + (7)(16)\right] = \pi(3)(16) = 48\pi$

27. (a) $V = \int_c^d 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dy = \int_1^2 2\pi y(y - 1)\, dy$

$= 2\pi\int_1^2 (y^2 - y)\, dy = 2\pi\left[\frac{y^3}{3} - \frac{y^2}{2}\right]_1^2$

$= 2\pi\left[\left(\frac{8}{3} - \frac{4}{2}\right) - \left(\frac{1}{3} - \frac{1}{2}\right)\right]$

$= 2\pi\left(\frac{7}{3} - 2 + \frac{1}{2}\right) = \frac{\pi}{3}(14 - 12 + 3) = \frac{5\pi}{3}$



(b) $V = \int_a^b 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dx$

$= \int_1^2 2\pi x(2 - x)\, dx = 2\pi\int_1^2 (2x - x^2)\, dx = 2\pi\left[x^2 - \frac{x^3}{3}\right]_1^2 = 2\pi\left[\left(4 - \frac{8}{3}\right) - \left(1 - \frac{1}{3}\right)\right]$

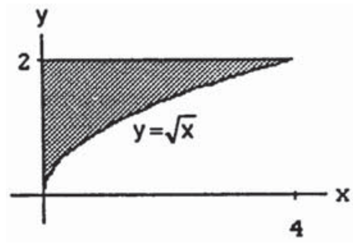
$= 2\pi\left[\left(\frac{12-8}{3}\right) - \left(\frac{3-1}{3}\right)\right] = 2\pi\left(\frac{4}{3} - \frac{2}{3}\right) = \frac{4\pi}{3}$

(c) $V = \int_a^b 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dx = \int_1^2 2\pi\left(\frac{10}{3} - x\right)(2 - x)\, dx = 2\pi\int_1^2 \left(\frac{20}{3} - \frac{16}{3}x + x^2\right) dx$

$= 2\pi\left[\frac{20}{3}x - \frac{8}{3}x^2 + \frac{1}{3}x^3\right]_1^2 = 2\pi\left[\left(\frac{40}{3} - \frac{32}{3} + \frac{8}{3}\right) - \left(\frac{20}{3} - \frac{8}{3} + \frac{1}{3}\right)\right] = 2\pi\left(\frac{3}{3}\right) = 2\pi$

(d) $V = \int_c^d 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dy = \int_1^2 2\pi(y - 1)(y - 1)\, dy = 2\pi\int_1^2 (y - 1)^2 = 2\pi\left[\frac{(y-1)^3}{3}\right]_1^2 = \frac{2\pi}{3}$

28. (a) $V = \int_c^d 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dy = \int_0^2 2\pi y(y^2 - 0)\, dy$
$= 2\pi \int_0^2 y^3\, dy = 2\pi \left[\frac{y^4}{4}\right]_0^2 = 2\pi \left(\frac{2^4}{4}\right) = 8\pi$

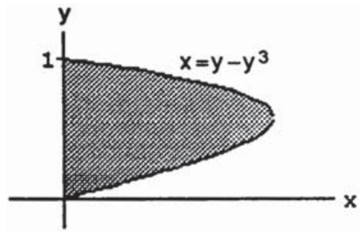


(b) $V = \int_a^b 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dx$
$= \int_0^4 2\pi x \left(2 - \sqrt{x}\right) dx = 2\pi \int_0^4 \left(2x - x^{3/2}\right) dx$
$= 2\pi \left[x^2 - \frac{2}{5} x^{5/2}\right]_0^4 = 2\pi \left(16 - \frac{2 \cdot 2^5}{5}\right)$
$= 2\pi \left(16 - \frac{64}{5}\right) = \frac{2\pi}{5}(80 - 64) = \frac{32\pi}{5}$

(c) $V = \int_a^b 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dx = \int_0^4 2\pi(4 - x)\left(2 - \sqrt{x}\right) dx = 2\pi \int_0^4 \left(8 - 4x^{1/2} - 2x + x^{3/2}\right) dx$
$= 2\pi \left[8x - \frac{8}{3} x^{3/2} - x^2 + \frac{2}{5} x^{5/2}\right]_0^4 = 2\pi \left(32 - \frac{64}{3} - 16 + \frac{64}{5}\right) = \frac{2\pi}{15}(240 - 320 + 192) = \frac{2\pi}{15}(112) = \frac{224\pi}{15}$

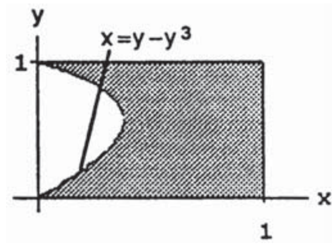
(d) $V = \int_c^d 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dy = \int_0^2 2\pi(2 - y)\left(y^2\right) dy = 2\pi \int_0^2 \left(2y^2 - y^3\right) dy = 2\pi \left[\frac{2}{3} y^3 - \frac{y^4}{4}\right]_0^2$
$= 2\pi \left(\frac{16}{3} - \frac{16}{4}\right) = \frac{32\pi}{12}(4 - 3) = \frac{8\pi}{3}$

29. (a) $V = \int_c^d 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dy = \int_0^1 2\pi y(y - y^3)\, dy$
$= \int_0^1 2\pi \left(y^2 - y^4\right) dy = 2\pi \left[\frac{y^3}{3} - \frac{y^5}{5}\right]_0^1 = 2\pi \left(\frac{1}{3} - \frac{1}{5}\right)$
$= \frac{4\pi}{15}$



(b) $V = \int_c^d 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dy$
$= \int_0^1 2\pi(1 - y)\left(y - y^3\right) dy$
$= 2\pi \int_0^1 \left(y - y^2 - y^3 + y^4\right) dy = 2\pi \left[\frac{y^2}{2} - \frac{y^3}{3} - \frac{y^4}{4} + \frac{y^5}{5}\right]_0^1 = 2\pi \left(\frac{1}{2} - \frac{1}{3} - \frac{1}{4} + \frac{1}{5}\right) = \frac{2\pi}{60}(30 - 20 - 15 + 12) = \frac{7\pi}{30}$

30. (a) $V = \int_c^d 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dy$
$= \int_0^1 2\pi y \left[1 - \left(y - y^3\right)\right] dy$
$= 2\pi \int_0^1 \left(y - y^2 + y^4\right) dy = 2\pi \left[\frac{y^2}{2} - \frac{y^3}{3} + \frac{y^5}{5}\right]_0^1$
$= 2\pi \left(\frac{1}{2} - \frac{1}{3} + \frac{1}{5}\right) = \frac{2\pi}{30}(15 - 10 + 6)$
$= \frac{11\pi}{15}$



(b) Use the washer method:
$V = \int_c^d \pi \left[R^2(y) - r^2(y)\right] dy = \int_0^1 \pi \left[1^2 - \left(y - y^3\right)^2\right] dy = \pi \int_0^1 \left(1 - y^2 - y^6 + 2y^4\right) dy = \pi \left[y - \frac{y^3}{3} - \frac{y^7}{7} + \frac{2y^5}{5}\right]_0^1$
$= \pi \left(1 - \frac{1}{3} - \frac{1}{7} + \frac{2}{5}\right) = \frac{\pi}{105}(105 - 35 - 15 + 42) = \frac{97\pi}{105}$

(c) Use the washer method:
$V = \int_c^d \pi \left[R^2(y) - r^2(y)\right] dy = \int_0^1 \pi \left[\left[1 - \left(y - y^3\right)\right]^2 - 0\right] dy = \pi \int_0^1 \left[1 - 2\left(y - y^3\right) + \left(y - y^3\right)^2\right] dy$
$= \pi \int_0^1 \left(1 + y^2 + y^6 - 2y + 2y^3 - 2y^4\right) dy = \pi \left[y + \frac{y^3}{3} + \frac{y^7}{7} - y^2 + \frac{y^4}{2} - \frac{2y^5}{5}\right]_0^1 = \pi \left(1 + \frac{1}{3} + \frac{1}{7} - 1 + \frac{1}{2} - \frac{2}{5}\right)$
$= \frac{\pi}{210}(70 + 30 + 105 - 2 \cdot 42) = \frac{121\pi}{210}$

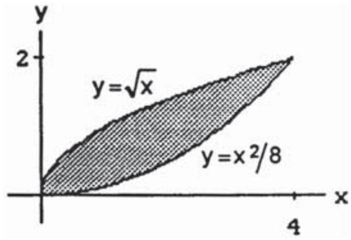
(d) $V = \int_c^d 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dy = \int_0^1 2\pi(1 - y)\left[1 - \left(y - y^3\right)\right] dy = 2\pi \int_0^1 (1 - y)\left(1 - y + y^3\right) dy$
$= 2\pi \int_0^1 \left(1 - y + y^3 - y + y^2 - y^4\right) dy = 2\pi \int_0^1 \left(1 - 2y + y^2 + y^3 - y^4\right) dy = 2\pi \left[y - y^2 + \frac{y^3}{3} + \frac{y^4}{4} - \frac{y^5}{5}\right]_0^1$
$= 2\pi \left(1 - 1 + \frac{1}{3} + \frac{1}{4} - \frac{1}{5}\right) = \frac{2\pi}{60}(20 + 15 - 12) = \frac{23\pi}{30}$

31. (a) $V = \int_c^d 2\pi \left(\substack{\text{shell}\\\text{radius}}\right)\left(\substack{\text{shell}\\\text{height}}\right) dy = \int_0^2 2\pi y \left(\sqrt{8y} - y^2\right) dy$

$= 2\pi \int_0^2 \left(2\sqrt{2}\, y^{3/2} - y^3\right) dy = 2\pi \left[\frac{4\sqrt{2}}{5} y^{5/2} - \frac{y^4}{4}\right]_0^2$

$= 2\pi \left(\frac{4\sqrt{2}\cdot\left(\sqrt{2}\right)^5}{5} - \frac{2^4}{4}\right) = 2\pi \left(\frac{4\cdot 2^3}{5} - \frac{4\cdot 4}{4}\right)$

$= 2\pi \cdot 4 \left(\frac{8}{5} - 1\right) = \frac{8\pi}{5}(8 - 5) = \frac{24\pi}{5}$



(b) $V = \int_a^b 2\pi \left(\substack{\text{shell}\\\text{radius}}\right)\left(\substack{\text{shell}\\\text{height}}\right) dx = \int_0^4 2\pi x \left(\sqrt{x} - \frac{x^2}{8}\right) dx = 2\pi \int_0^4 \left(x^{3/2} - \frac{x^3}{8}\right) dx = 2\pi \left[\frac{2}{5} x^{5/2} - \frac{x^4}{32}\right]_0^4$

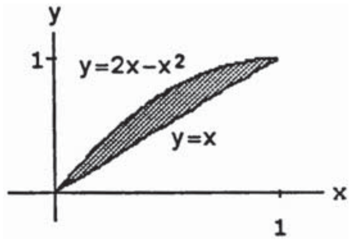
$= 2\pi \left(\frac{2\cdot 2^5}{5} - \frac{4^4}{32}\right) = 2\pi \left(\frac{2^6}{5} - \frac{2^8}{32}\right) = \frac{\pi\cdot 2^7}{160}(32 - 20) = \frac{\pi\cdot 2^9\cdot 3}{160} = \frac{\pi\cdot 2^4\cdot 3}{5} = \frac{48\pi}{5}$

32. (a) $V = \int_a^b 2\pi \left(\substack{\text{shell}\\\text{radius}}\right)\left(\substack{\text{shell}\\\text{height}}\right) dx$

$= \int_0^1 2\pi x \left[\left(2x - x^2\right) - x\right] dx$

$= 2\pi \int_0^1 x\left(x - x^2\right) dx = 2\pi \int_0^1 \left(x^2 - x^3\right) dx$

$= 2\pi \left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = 2\pi \left(\frac{1}{3} - \frac{1}{4}\right) = \frac{\pi}{6}$



(b) $V = \int_a^b 2\pi \left(\substack{\text{shell}\\\text{radius}}\right)\left(\substack{\text{shell}\\\text{height}}\right) dx = \int_0^1 2\pi(1 - x)\left[\left(2x - x^2\right) - x\right] dx = 2\pi \int_0^1 (1 - x)\left(x - x^2\right) dx$

$= 2\pi \int_0^1 \left(x - 2x^2 + x^3\right) dx = 2\pi \left[\frac{x^2}{2} - \frac{2}{3}x^3 + \frac{x^4}{4}\right]_0^1 = 2\pi \left(\frac{1}{2} - \frac{2}{3} + \frac{1}{4}\right) = \frac{2\pi}{12}(6 - 8 + 3) = \frac{\pi}{6}$

33. (a) $V = \int_a^b \pi \left[R^2(x) - r^2(x)\right] dx = \pi \int_{1/16}^1 \left(x^{-1/2} - 1\right) dx$

$= \pi \left[2x^{1/2} - x\right]_{1/16}^1 = \pi \left[(2 - 1) - \left(2\cdot\frac{1}{4} - \frac{1}{16}\right)\right]$

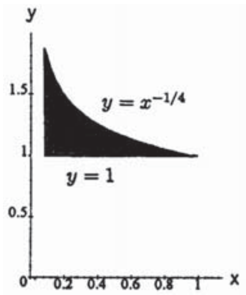
$= \pi\left(1 - \frac{7}{16}\right) = \frac{9\pi}{16}$

(b) $V = \int_a^b 2\pi \left(\substack{\text{shell}\\\text{radius}}\right)\left(\substack{\text{shell}\\\text{height}}\right) dy = \int_1^2 2\pi y \left(\frac{1}{y^4} - \frac{1}{16}\right) dy$

$= 2\pi \int_1^2 \left(y^{-3} - \frac{y}{16}\right) dy = 2\pi \left[-\frac{1}{2} y^{-2} - \frac{y^2}{32}\right]_1^2$

$= 2\pi \left[\left(-\frac{1}{8} - \frac{1}{8}\right) - \left(-\frac{1}{2} - \frac{1}{32}\right)\right] = 2\pi \left(\frac{1}{4} + \frac{1}{32}\right)$

$= \frac{2\pi}{32}(8 + 1) = \frac{9\pi}{16}$



34. (a) $V = \int_c^d \pi \left[R^2(y) - r^2(y)\right] dy = \int_1^2 \pi \left(\frac{1}{y^4} - \frac{1}{16}\right) dy$

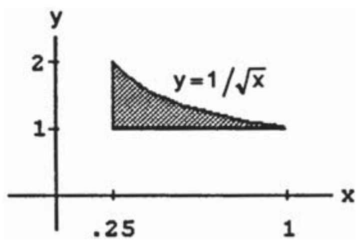
$= \pi \left[-\frac{1}{3} y^{-3} - \frac{y}{16}\right]_1^2 = \pi \left[\left(-\frac{1}{24} - \frac{1}{8}\right) - \left(-\frac{1}{3} - \frac{1}{16}\right)\right]$

$= \frac{\pi}{48}(-2 - 6 + 16 + 3) = \frac{11\pi}{48}$

(b) $V = \int_a^b 2\pi \left(\substack{\text{shell}\\\text{radius}}\right)\left(\substack{\text{shell}\\\text{height}}\right) dx = \int_{1/4}^1 2\pi x \left(\frac{1}{\sqrt{x}} - 1\right) dx$

$= 2\pi \int_{1/4}^1 \left(x^{1/2} - x\right) dx = 2\pi \left[\frac{2}{3} x^{3/2} - \frac{x^2}{2}\right]_{1/4}^1$

$= 2\pi \left[\left(\frac{2}{3} - \frac{1}{2}\right) - \left(\frac{2}{3}\cdot\frac{1}{8} - \frac{1}{32}\right)\right] = \pi \left(\frac{4}{3} - 1 - \frac{1}{6} + \frac{1}{16}\right) = \frac{\pi}{48}(4\cdot 16 - 48 - 8 + 3) = \frac{11\pi}{48}$



35. (a) *Disk*: $V = V_1 - V_2$

$V_1 = \int_{a_1}^{b_1} \pi[R_1(x)]^2\, dx$ and $V_2 = \int_{a_2}^{b_2} \pi[R_2(x)]^2$ with $R_1(x) = \sqrt{\frac{x+2}{3}}$ and $R_2(x) = \sqrt{x}$,

$a_1 = -2,\ b_1 = 1;\ a_2 = 0,\ b_2 = 1 \Rightarrow$ two integrals are required

(b) *Washer*: $V = V_1 - V_2$

$V_1 = \int_{a_1}^{b_1} \pi\left([R_1(x)]^2 - [r_1(x)]^2\right) dx$ with $R_1(x) = \sqrt{\frac{x+2}{3}}$ and $r_1(x) = 0$; $a_1 = -2$ and $b_1 = 0$;